SOLUTION OF LAMINAR COMPRESSIBLE BOUNDARY LAYER EQUATION BY PARAMETRIC DIFFERENTIATION METHOD

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FEBRUARY, 1970

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A Thesis Submitted
In Partial Fulfilment of the Requirement
for the Degree of
MASTER OF TECHNOLOGY

BY

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to the

DEPARTMENT OF MECHANICAL ENGINEERING

A C K N O W L E D G M E N T

The author recognises the invaluable suggestions and the precious help of Dr. Oberai in selection and completion of the problem.

C E R T I F I C A T E.

This is to certify that the present work has been carried out under my supervision and has not been submitted elsewhere for a degree.

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ABSTRACT

In the present work, two-dimensional laminar compressible boundary layer problem under the assumption of either Prandtl number equal to unity or that of low mach number, has been solved numerically using Parametric Differentiation Method. At the end, a general program based on Fortran IV has been developed by which through the use of the initial value obtained by the Parametric Differentiation Method as starting guess, the original non-linear equation can be solved to any degree of accuracy.

CONTENTS

CHAPTER 1

INTRODUCTION

The equations governing the two dimensional compressible laminar boundary layer are coupled in addition to being non-linear. Except for the special case of a flow with Prandtl number unity over an insulated surface, these equations are difficult to solve.

With no standard method can the solutions of these two point boundary value problems be expressed in a closed form. The following two methods are generally resorted to for solving these equations.

1. Forward integration
2. Integration by successive approximation

Both of these methods are highly iterative and a good amount of guess work has to be done for rapid convergence to the solution. The aim of the present work is to dispense with the guess work and to solve the equations by parametric differentiation (Ref. 5). As we shall see, this method reduces the equation to be solved to a linear system which, though of higher order, can be solved by a predetermined number of iterations.

Satisfactory results obtained in the case of the compressible boundary layer encouraged us to develope a general

program which can handle all those ordinary differential equations which can be solved by parametric differentiation method. In the last chapter we have presented this general program. Due to the flexibilities incorporated in the program, it can handle, in addition to the reduced linear system of equation, the parent non-linear system of equations. This flexibility is used when a high accuracy is needed. Here the solution is obtained in two steps in the first step, the usual solution is obtained using parametric differentiation method. In the second step, the nonlinear equations are directly solved. Here the missing initial boundary conditions are taken from the solutions obtained in the first step. As initial boundary condition taken are very close to the two one. This method converges very rapidly in a few iterations to the true solution .

CHAPTER 2

FORMULATION OF THE PROBLEM (Ref.4)

2.1 Stewartson's Equation:

Steady two-dimensional compressible laminar boundary layer for perfect gas are given by

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad \text{(Cont. Eqn.)} \qquad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \qquad \text{(Mom. Eqn.)} \qquad (2)$$

$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = u \frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\frac{\partial u}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2$$

(Energy Eqn.)(3)

where,

p = pressure

ρ = density

u = longitudinal velocity

v = transverse velocity

x = longitudinal coordinate

y = transverse coordinate

h = enthalpy

μ = coefficient of viscosity

Pr = Prandtl Number

The viscosity law to be assumed is

$$\frac{\mu}{\mu_o} = \lambda \frac{t}{t_o} \qquad (4)$$

The constant λ is used to match the viscosity with the Sutherland value (Ref. 3) at a desired station. If this station is taken to be the surface assumed to be at constant temperature the result is

$$\lambda = \sqrt{(t_w/t_o)} \quad \left(\frac{t_o + K_{su}}{t_w + K_{su}} \right) \tag{5}$$

where

t_w = wall temperature

t_o = free strain stagnation temperature

K_{su} = Sutherland's constant

2.2 Stewartson's Transformation:

Velocities in the equation of motion (1) to (3) can be expressed in terms of derivatives of stream function as

$$\begin{aligned} \Psi_y &= (\rho u/\rho_o) \\ \Psi_x &= (\rho v/\rho_o) \end{aligned} \tag{6}$$

Now introducing transformation

$$\begin{aligned} X &= \int_o^X \frac{p_e}{p_o} \cdot \frac{a_e}{a_o} \, dx \\ Y &= \frac{a_e}{a_o} \int_o^Y \frac{\rho}{\rho_o} \, dy \end{aligned} \tag{7}$$

(Subscript e refers to local condition at the outer edge of the boundary layer (external). The subscript o refers to the free-stream stagnation values.) Where a is sonic velocity.

We get the following equations:

$$U_X + V_Y = 0 \tag{8}$$

$$UU_X + VU_Y = U_e U_{eX} S + \nu_o U_{YY}$$

$$US_X + VS_Y = \nu_o \left(\frac{S_{YY}}{Pr} - \right.$$

$$\left. - \frac{1-Pr}{Pr} \left\{ \frac{((\gamma-1)/2)M_e^2}{1+((\gamma-1)/2)M_e^2} \right\} \left(\frac{U}{U_e}\right)^2 \right)_{YY} \tag{9}$$

where

γ = ratio of specific heats

ν = μ/ρ

$$s = h_s/h_o \tag{10}$$

a is local sonic velocity.

s is enthalpy function

and h_s is local stagnation enthalpy.

Transformed velocities U, V are related to stream function by the following equations:

$$U = -\psi_Y , \qquad V = \psi_X$$

The transformed longitudinal velocity U is related to the longitudinal velocity in physical plane by

$$U = (a_o/a_e)\, u$$

The boundary conditions applicable to eqn. (7) to (9) are

$$U(X, 0) = 0$$

$$V(X, 0) = 0$$

$$s(X, 0) = s_w \quad \text{or} \quad \frac{\partial s}{\partial y}(X, 0) = \left(\frac{\partial s}{\partial y}\right)_w \tag{11}$$

$$\lim_{Y\to\infty} s = 0$$

Now under the transformation

$$\psi = f(\eta)\sqrt{(2\nu_o U_e X)/(m+1)}$$
$$\eta = Y\sqrt{0.5\,(m+1)\,U_e/(\nu_o X)} \qquad (12)$$

The above system of partial equations is reduced to following ordinary diffrential equation

$$f''' + ff'' = \beta(f'^2 - S)$$
$$S'' + Pr\,f\,S' = (1-Pr)\left[\frac{(\gamma-1)M_e^2}{1+((\gamma-1)/2)M_e^2}\right](f'f''' + f''^2) \qquad (1$$

The pressure gradient parameter β is defined as

$$\beta = \frac{2m}{m+1}$$

where m is given by

$$U_e = CX^m$$

where C is constant.

The velocity ratio $U/U_e = u/u_e = f'$

In above equation prime denotes differentiation with respect to η.

The boundary conditions are

$$f(0) = f'(0) = 0$$
$$S(0) = S_w$$
$$\lim_{\eta\to\infty} f' = 1 \qquad (14)$$

Now in the energy equation, the right handside is not functionally consistent with arbitrary Mach no. M_e which is a function of X. Hence to be consistent with left handside right handside of energy equation must be either constant or function of η. Now this is true for following cases

1. When external Mach number is constant as is in the case of flate plate.
2. External Mach number is negligibly small.
3. Prandtl number is equal to unity
4. Specific heat ratio is equal to unity. In reality γ cannot be equal to uhity.
5. Mach number is very high ($Me \to \infty$) as in this case right hand side will be equal to 2.

Here problem will be treated assuming either Prandtl number is equal to 1 or Mach number is very small.

Hence system of equations to be solved are

$$f''' + ff'' = \beta\,(f'^2 - s) \tag{15}$$

$$s'' + fs'\,Pr = 0 \tag{16}$$

with boundary conditions (14)

CHAPTER 3

METHOD OF SOLUTION

3.1 Reduction of Nonlinear Equations to Linear Form:

Essence of method based on parametric differentiation is to differentiate the original equation with respect to a parameter, and to solve the resulting linear equation.

Now to apply this method we have to know the solution of the equation for one value of parameter and then we march forward for other values of the parameter.

Now differentiating equations (15) and (16) with respect to β we get,

$$G''' + Gf'' + fG'' = (f'^2 - S) + \beta(2f'G' - T) \tag{17}$$

$$T'' + Pr * (fT' + GS') = 0 \tag{18}$$

with boundary conditions

$$G'(0) = G(0) = 0, \quad T(0) = 0, \; T(\infty) = 0$$

where

$$\begin{aligned} df/d\beta &= G \\ dS/d\beta &= T \end{aligned} \tag{19}$$

where prime denotes differentiation with respect to η

Equations (17) and (18) are linear in G and T.

Now after solving for f and S for initial value of β, we substitute for f and S in linear equation (17) and (18) and evaluate G and T. From these values of G and T, we

solve for f and s from equations (19). This cycle is repeated till β reaches the desired value. Both the systems of equations (17, 18) and (19) are solved using Runge-Kutta method with proper increment in η and β respectively.

Initially such a value of parameter is chosen that at this value either the solution is known or it can be obtained very easily.

Here the initial value of parameter β is chosen as zero. For this value of β, the equations (15, 16) are reduced to

$$f''' + ff'' = 0 \tag{20}$$

$$s'' + Prfs' = 0 \tag{21}$$

with boundary conditions (14).

Equation (20) is Blasius equation whose solution is known or can be obtained very easily.

Solution for other values of β from that at the initial value of β is obtained following the method outlined above.

In actual numerical solution, increment in η has been taken equal to .05 and that in β equal to 0.1 for favourable pressure gradient and -0.05 for adverse pressure gradient. Solutions were obtained for various Prandtl numbers and wall temperatures at different values of β.

We have seen equations to be solved for β equal to zero are uncoupled and simple but still it is nonlinear. We can further simplify the problem by further use of parametric

differentiation method. In Blasius equation there is no parameter but we can artificially introduce a parameter as shown below.

Let us consider the equation,

$$f''' + (1-P)f'' + P\,ff'' = 0 \tag{22}$$

with boundary condition

$$f(0) = f'(0) = 0$$
$$f'(\infty) = 1$$

Above equation for P equal to one is reduced to Blasius equation. For P equal to zero equation is reduced to,

$$f''' + f'' = 0 \tag{23}$$

with boundary condition

$$f(0) = f'(0) = 0 \quad f'(\infty) = 1.$$

Solution of equation (23) is very simple and is given by

$$f = \eta + e^{-\eta} - 1$$

Now differentiating equation (22) with respect to P, we get,

$$W''' + (1-P)W'' + f'' + ff'' + PWf'' + PfW'' = 0 \tag{24}$$

with boundary conditions

$$\begin{aligned} W(0) &= W'(0) = 0 \\ W(\infty) &= 1 \end{aligned} \tag{25}$$

Where,

$$df/dP = W \tag{26}$$

Equations (22, 24 and 26) are solved in the same way as equations (17, 18 and 19).

Equation (23) has been solved with increment in h equal to 0.05 and that in P equal to 0.05. Results obtained tally fairly with the standard result. Results are shown in table (1) on the next page.

Similarly initial solution for both the equations (15, 16) can be obtained by assuming suitable equations. This has been illustrated in Chapter 5.

Table 1

Solution of Equation (22) by Parametric Differentiation for Pr = 1. (Numerical solution for Blasius Equation)

ETA (η)	F	ETA (η)	F
0	0	3.8	2.583
0.2	.0094	4.0	2.782
0.4	.0375	4.2	2.982
0.6	.0843	4.4	3.181
0.8	.1495	4.6	3.381
1.0	.2328	4.8	3.581
1.2	.3333	5.0	3.781
1.4	.4503	5.2	3.981
1.6	.5824	5.4	4.181
1.8	.7282	5.6	4.380
2.0	.8860	5.8	4.580
2.2	1.054	6.0	4.780
2.4	1.231	6.2	4.980
2.6	1.414	6.4	5.180
2.8	1.602	6.6	5.380
3.0	1.794	6.8	5.580
3.2	1.989	7.0	5.780
3.4	2.186	7.2	5.980
3.6	2.384	7.4	6.178

CHAPTER 4

RESULTS OBTAINED

4.1 Velocity and Enthalpy Function:

The velocity and enthalpy functions are presented in tabular form.

The distance y normal to the surface in the physical plane is related to the similarity variable η through equation (6) and (12) as (Ref. 4).

$$y = \frac{p_o}{p_e} \cdot \frac{a_o}{a_e} \sqrt{\frac{2}{m+1} \frac{\nu_o X}{U_e}} \int_0^{\eta} \frac{t}{t_o}\, d\eta$$

where

$$t/t_o = \left(1 + \frac{\gamma-1}{2} M_e^2\right) S - \frac{\gamma-1}{2} M_e^2 \, f'^2 \tag{27}$$

4.2 Integral Thicknesses:

The boundary layer integral thicknesses **in the** transformed plane are defined by the following relations (Ref.4)

Displacement thickness:

$$\frac{\delta^*_{tr}}{X} \sqrt{\frac{m+1}{2} \frac{U_e X}{\nu_o}} = \int_0^{\infty} (S - f')\, d\eta \tag{28}$$

Momentum thickness:

$$\frac{\theta_{tr}}{X} \sqrt{\frac{m+1}{2} \frac{U_e X}{\nu_o}} = \int_0^{\infty} f'(1 - f')\, d\eta \tag{29}$$

Thermal thickness:

$$\frac{\epsilon}{X} \quad \frac{m+1}{2} \sqrt{\frac{U_e X}{\nu_o}} = \int_0^{\infty} S\, d\eta \tag{30}$$

Convection thickness:

$$\frac{E}{X} = \frac{m+1}{2} \; \frac{U_e X}{\nu_o} = \int_o^{\infty} (S-1) \, f' d = -S_w' \qquad (31)$$

Numerical values of these thicknesses are evaluated for different values of β, Pr and S_w.

4.3 Shear and skin friction:

The quantity that is of primary interest in boundary layer calculation is the shear stress at wall τ_w which in nondimensional form can be given as (Ref. 4)

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho_w u_e^2} = f_w'' \, (2\lambda(1+S_w)) \sqrt{\frac{m+1}{2} \; \frac{\nu_o}{U_e X}} \qquad (32$$

which can be rewritten as

$$\frac{C_f \sqrt{Re_w}}{2} = f_w'' \sqrt{\frac{m+1}{2} \; \frac{d \ln X}{d \ln x}} \qquad (33)$$

where C_f is called local skin friction coefficient.

$$Re_w = \frac{U_e x}{\nu_w}$$

w subscript denotes properties at wall.

4.4 Heat Transfer:

$S' = ds/d\eta$ at wall corresponds to heat transfer across boundary layer. This is related to stagnation enthalpy derivative in the physical plane by

$$\frac{\partial}{\partial y} \left(\frac{h_s}{h}\right) = \left(\frac{\rho a_e}{\rho a} \sqrt{\frac{m+1}{2} \; \frac{U_e}{\nu X}} \right) S' \qquad (34)$$

A non-dimensional quantity in connection with heat transfer can be introduced (Ref. 4) as

$$Nu = \frac{x(\partial t/\partial y)_w}{t_o - t_w} = \left(-\frac{S_w'}{S_w - 1}\right) \sqrt{Re_w} \sqrt{\frac{m+1}{2} \frac{d \ln X}{d \ln x}} \quad (35)$$

Reynolds analogy:

A simple modified Reynold analogy can be defined as

$$\frac{C_f \; Re_w}{Nu} = \frac{2f_w''}{(-S_w' / S_w - 1)} \quad (36)$$

This quantity is the reciprocal of usual Reynold analogy quantity. This factor is tabulated for different values of β at different values of Pr and Sw .

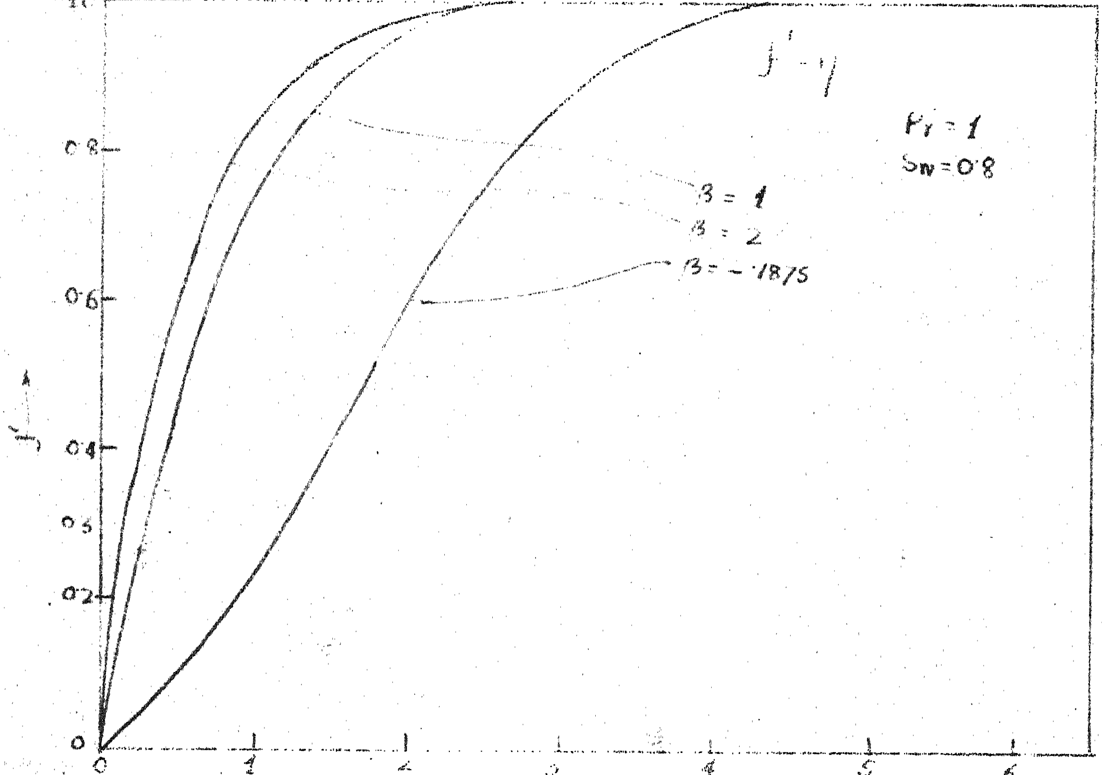
Numerical results are given in Appendix 1. Computer program for this is given in Appendix 2.

4.4 Some Comments on the Result:

In Fig. (1, 2, 3) velocity profiles have been shown. We observe that

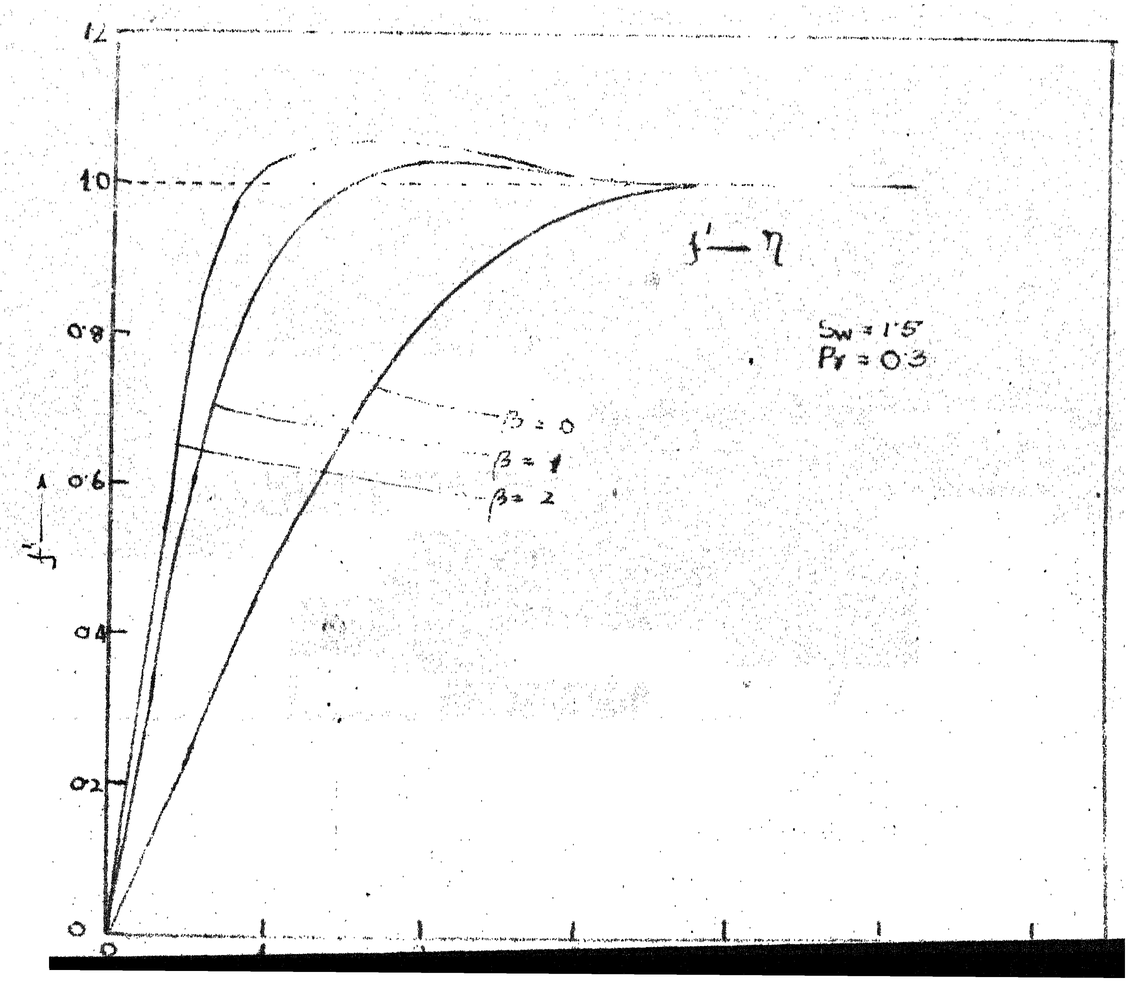
i) for a given wall temperature and Prandtl number, the initial slope of velocity increases with the pressure gradient parameter (Fig. 7 also).

ii) for adverse pressure gradient, an inflexion occurs within the boundary layer that moves outwards with increase in magnitude of negative pressure gradient parameter.

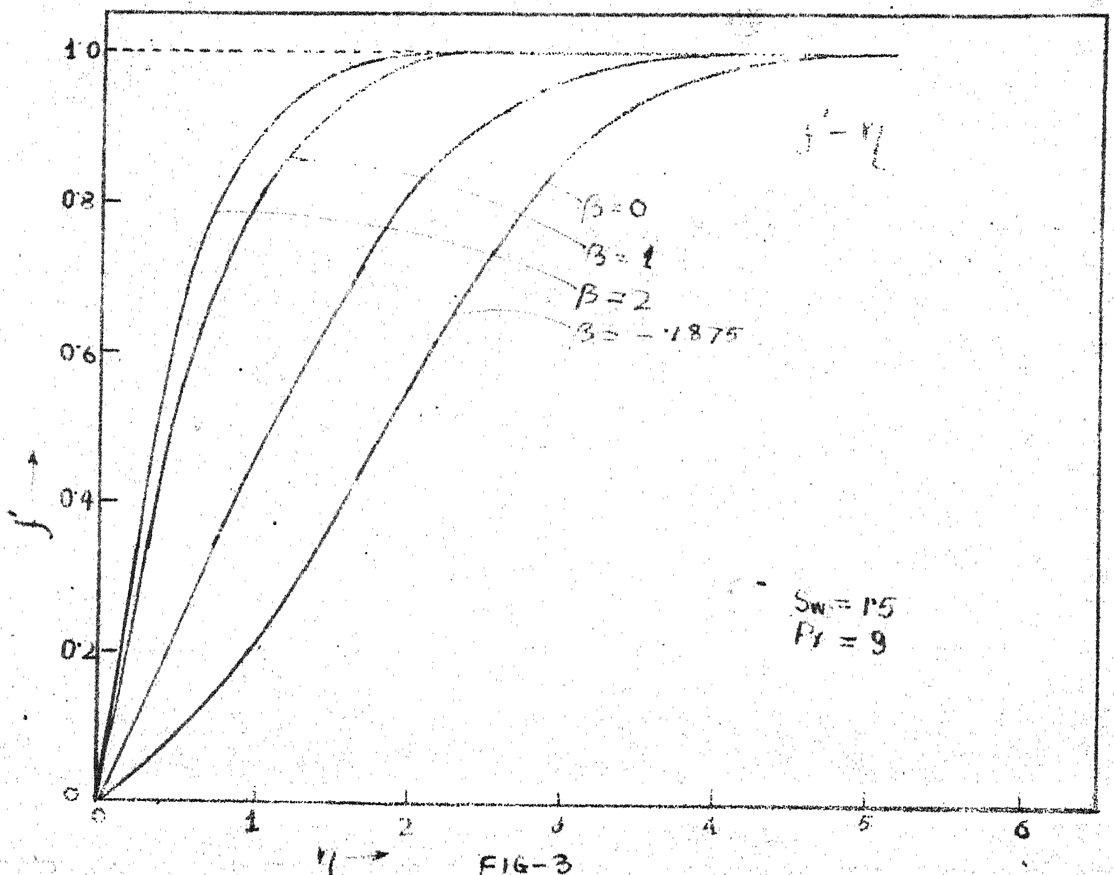


f' – η
Pr = 1
Sw = 0.8
β = 1
β = 2
β = −.1875
0.8
0.6
0.4
0.2
0
f'
0
1
2
4
5
6
η

FIG - 1

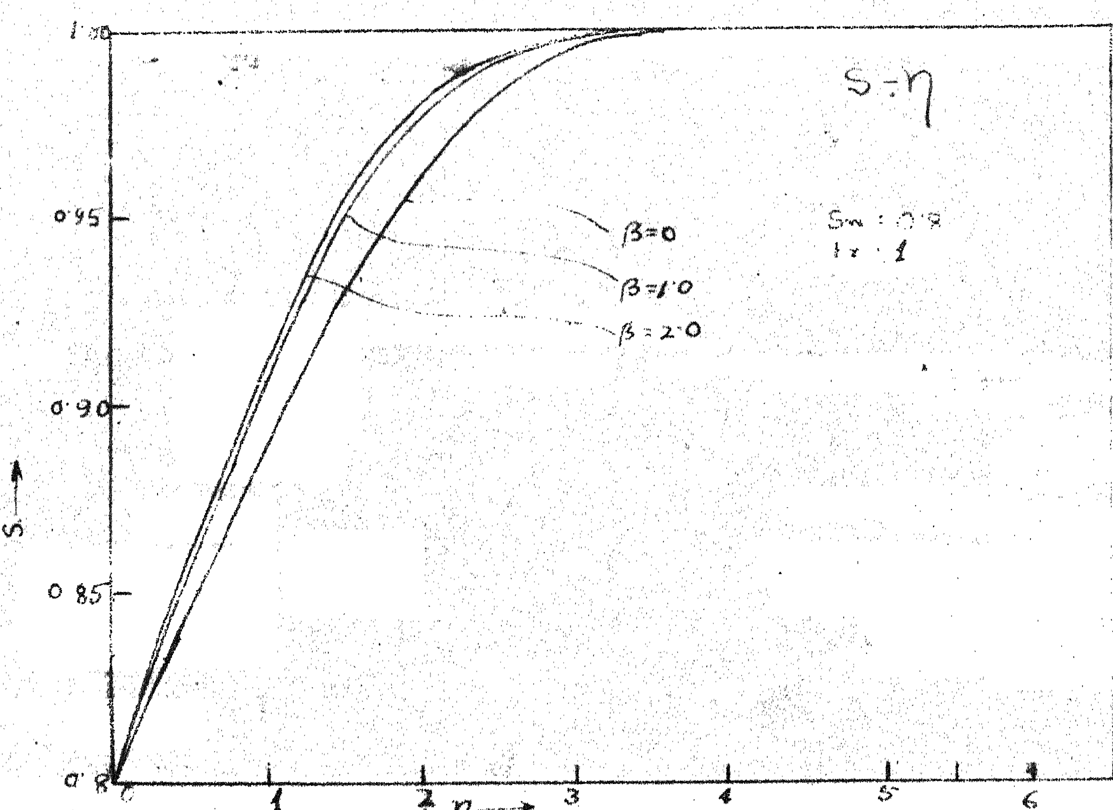


1.2
1.0
0.8
0.6
0.4
0.2
0
f'
f' — η
Sw = 1.5
Pr = 0.3
β = 0
β = 1
β = 2
0



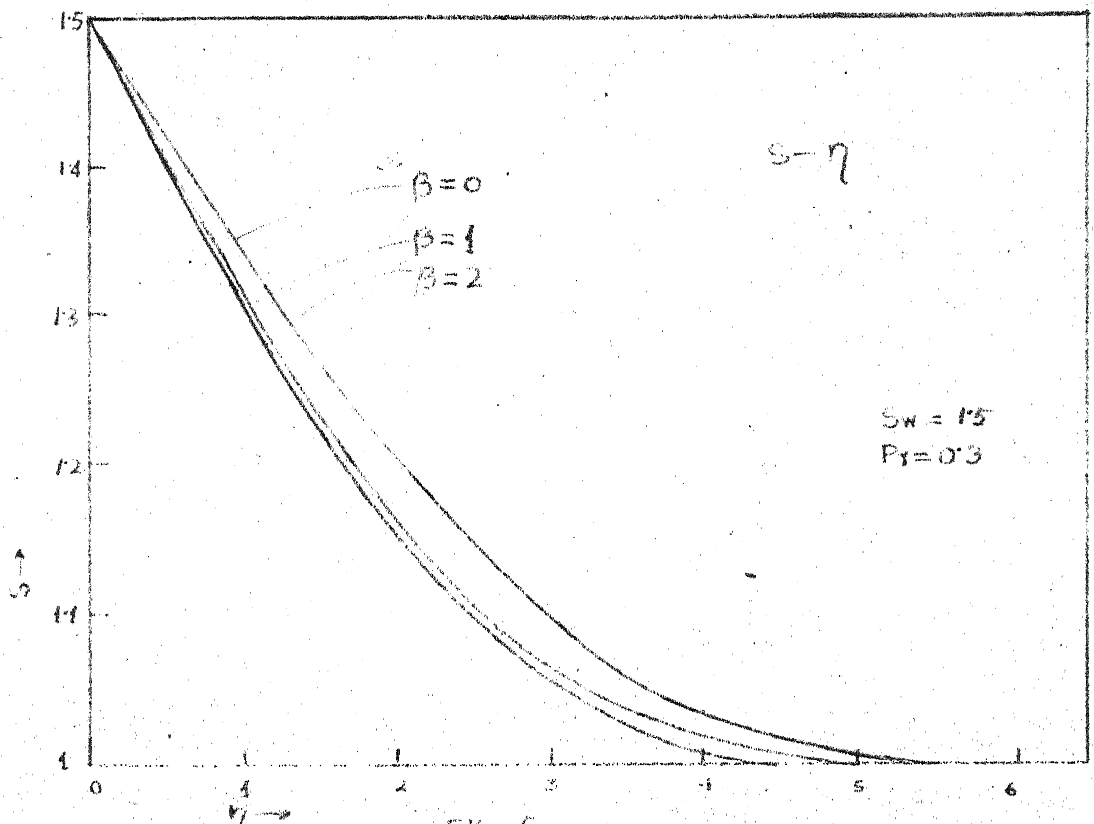
1·0
0·8
0·6
0·4
0·2
0
f'
0
1
2
3
4
5
6
η
$f' - \eta$
$\beta = 0$
$\beta = 1$
$\beta = 2$
$\beta = -\cdot 1875$
$S_w = 1\cdot 5$
$Pr = 9$

FIG-3



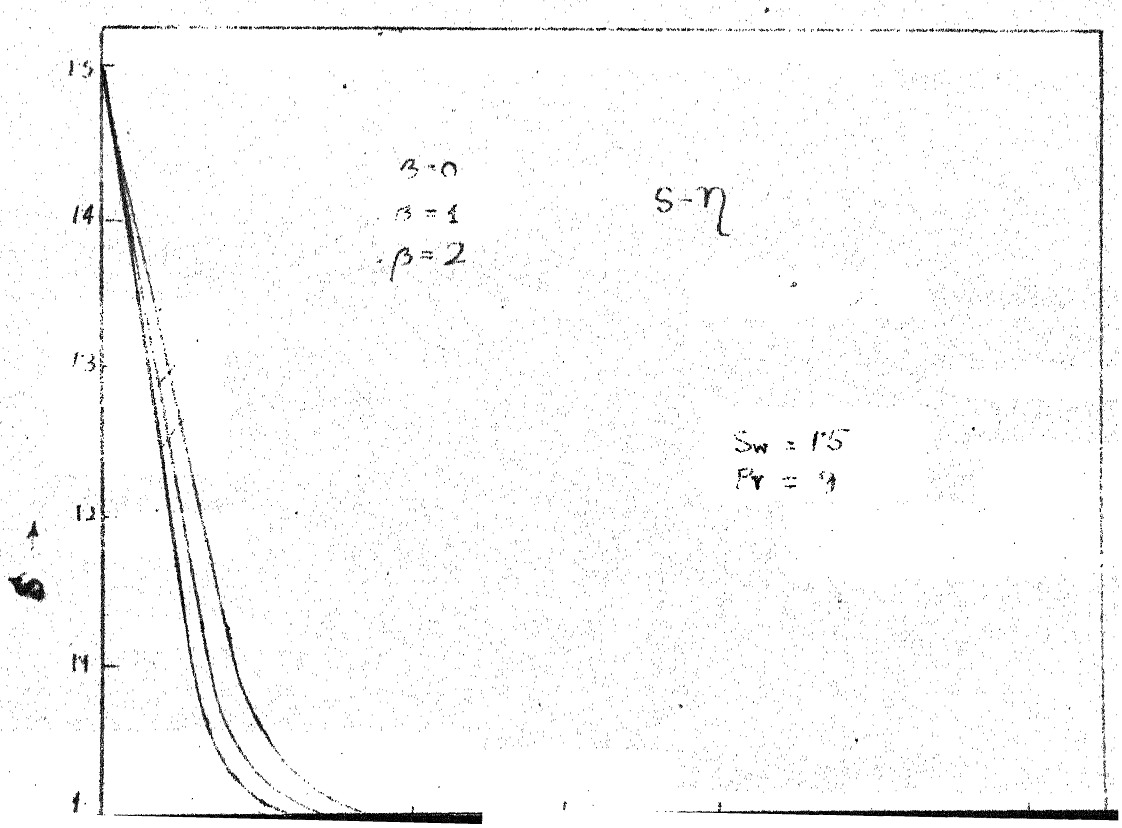
1·00
0·95
0·90
0·85
0·8
S
0
1
2
3
4
5
6
η
$S - \eta$
$\beta = 0$
$\beta = 1\cdot 0$
$\beta = 2\cdot 0$
$S_w = 0\cdot 8$

FIG 4

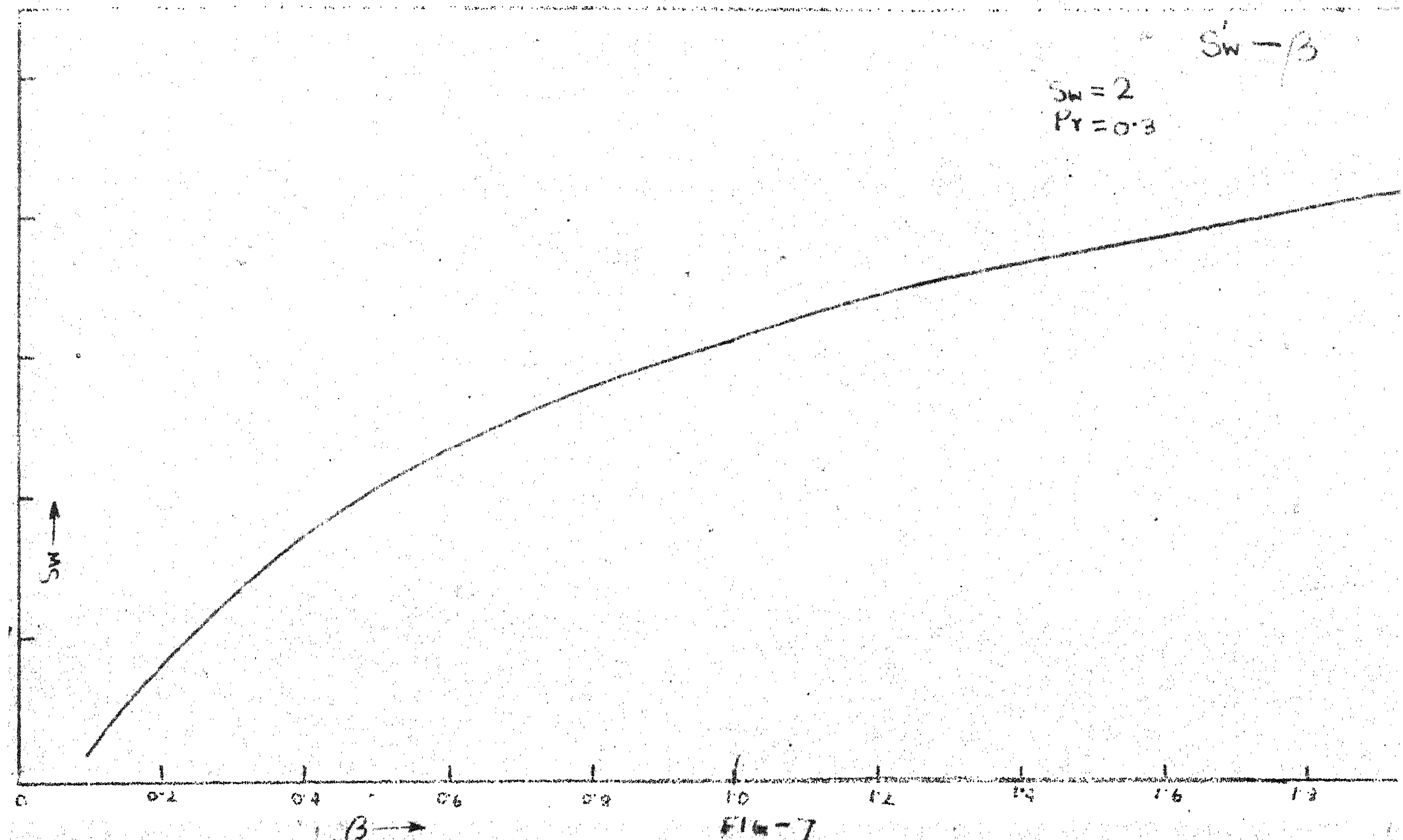


1·5
1·4
1·3
1·2
1·1
1
0
1
2
3
4
5
6
$\beta=0$
$\beta=1$
$\beta=2$
S-η
Sw = 1·5
Pr = 0·3
S
η

FIG-5

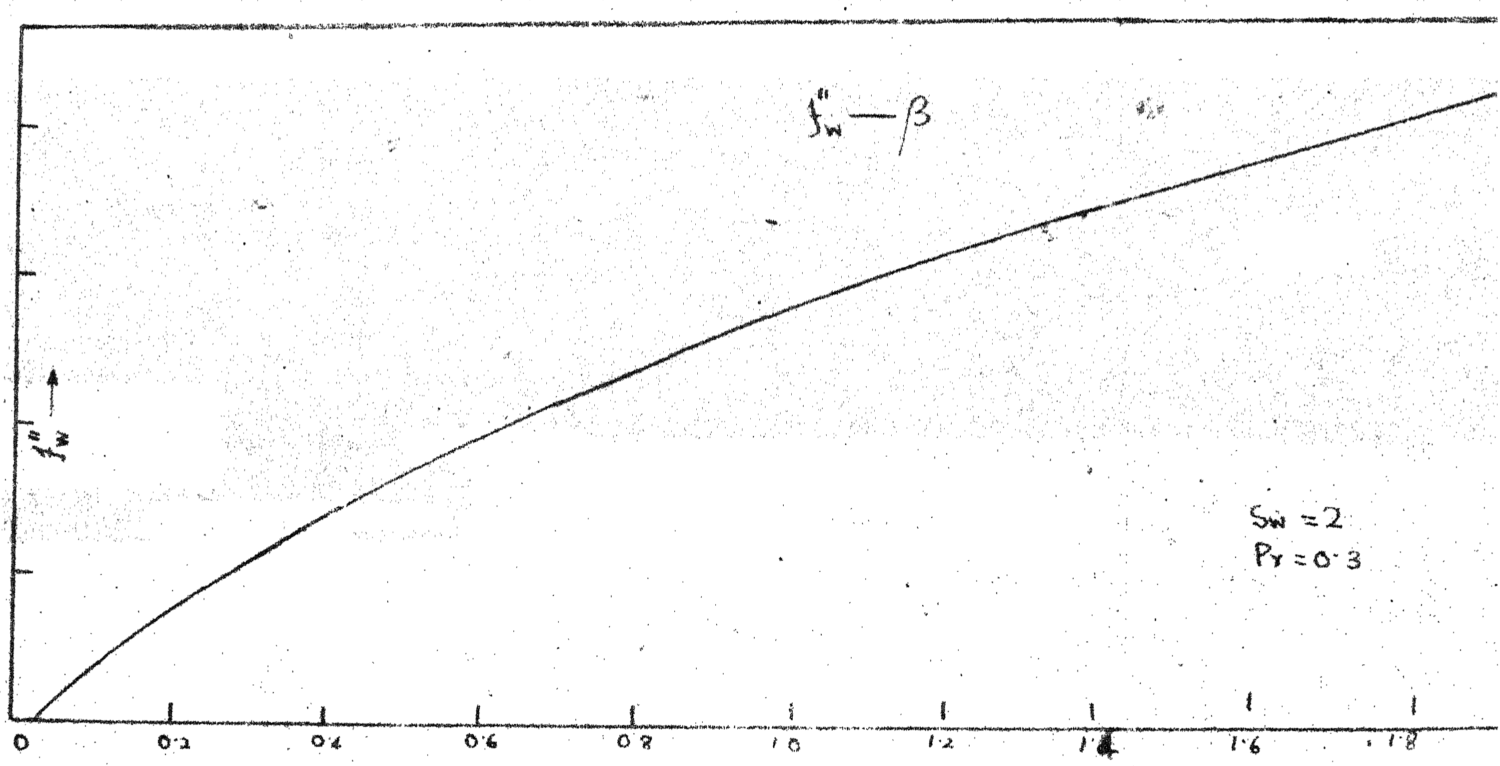


1·5
1·4
1·3
1·2
1·1
1
$\beta=0$
$\beta=1$
$\beta=2$
S-η
Sw = 1·5
Pr = 9
S



S'w — β
Sw = 2
Pr = 0·3
Sw →
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4
1·6
1·8
β →

FIG - 7



f''w — β
f''w →
Sw = 2
Pr = 0·3
0
0·2
0·4
0·6
0·8
1·0
1·2
1·4
1·6
1·8
β →

FIG - 8

iii) for $Sw > 1$, (or for heated surface) under favourable pressure gradient a velocity overshoot (Fig. 2) occurs within the boundary layer. This overshoot increases as the pressure gradient parameter becames more favourable. The over shoot takes place due to the fact that when wall is heated in a favourable pressure gradient, the density in certain region of boundary layer is lowered so that in spite of viscous drag, the flow there is accelerated more than the external flow by the pressure force.

iv) velocity boundary layer increases with increase in Prandtl number.

v) In Fig. (3,4,5) stagnation temperature profiles have been drawn. We see for a given Prandtl no. and specified wall temperature profile vary very slowly for different pressure gradient parameters. This variation becomes prominent with increase in the wall temperature. Like initial velocity gradient, slope of initial temperature gradient at wall increases as the pressure gradient becomes more favourable (Fig. 8 also).

vi) Thermal boundary layer thickness decreases with increase in Prandtl number.

By inspecting tabulated results, we observe that for low wall temperature in favourable pressure gradient displacement thickness is negative. This is due to the fact that the fluid in contact with the cold wall has higher density than that in the external flow. Hence more fluid /area

CHAPTER 5

SOME DETAILS OF COMPUTER PROGRAMMING

5.1 Iteration Scheme Used for Satisfying Boundary Conditions

Before explaining about the iteration scheme, we will mention one of the important properties of the ordinary linear differential equation.

Let the system of linear differential equation be given by

$$L(D) = 0$$

Now this can be reduced to a set of linear differential equation of order one, number of equations of order one being equal to the sum of the orders of the original ~~system of~~ equations. Any derivative of any dependent variable in the original equations will correspond to one dependent variable in this new set of equations of order one.

Let x_1, x_2, x_3, ... x_k be dependent variables in the new set of equations. Let $x_i(0)$, $x_b(0)$,... $x_p(0)$ are missing initial conditions whose value we have to know for solving the problem as initial value problem.

Let the conditions to be satisfied at the end point be

$$x_a(R) = FN(1)$$
$$x_c(R) = FN(2)$$

----

where FN(1), FN(2), ... FN(n) are values of variables x_a, x_c etc. to be satisfied at the end point R. Let us relebel the missing initial conditions as G(1), G(2), ...G(n).

Thus

$$
\begin{aligned}
G(1) &= x_i(0) \\
G(2) &= x_b(0) \\
&\text{-----------} \\
&\text{-----------} \\
G(n) &= x_p(0)
\end{aligned}
$$

Similarly, x_a, x_c, ... x_p are relebelled as

$$
\begin{aligned}
Y(1) &= x_a(R) = FN(1) \\
Y(2) &= x_c(R) = FN(2) \\
&\text{-----------------} \\
&\text{-----------------} \\
Y(n) &= x_q(R) = FN(n)
\end{aligned}
$$

Let $G(i) = C(i)$, for $i = 1,---n$

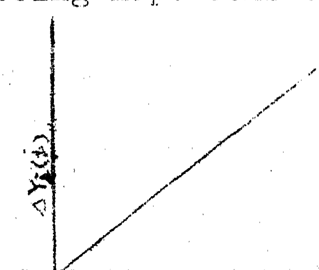
where C(i) is a value assumed for G(i).

Now for linear equation it is well known that if all the initial conditions are kept fixed, and an increment is given to the initial value of a variable then

$$\Delta Y_i(j) = C_{ji}\, \Delta G(i) \tag{37}$$

where $\Delta Y_i(j)$ is increment in Y(j) due to the increment G(i) in G(i), all other initial values of variables being kept constant C_{ji} is a constant.

Graphically it is shown in the adjacent figure



Now if there are increments x_1, --- x_n in G(1), G(2)---G(n) then total increment in Y(j) will be given by

$$\Delta Y(j) = \sum_{i=1}^{n} \Delta Y_i(j) = \sum_{i=1}^{n} C_{ji}\, x_i \tag{38}$$

Now we can treat the problem inversely in the following manner. Giving different values to initial conditions, we can calculate coefficients like C_{ji} from (37).

Now ~~if~~ $\Delta Y(j)$ is departure of Y(j) from the prescribed FN(j), when initial values are assumed as

$$G(i) = C(i)\ ,\ i = 1, \text{---}\ n$$

Now for a given $\Delta Y(j)$, we can solve for x_i from the simultaneous algebraic equations (38). After obtaining x_i, correct initial conditions which will satisfy the given boundary conditions will be given by

$$G(i) = C(i) + x(i),\quad i = 1, \text{---}\ n \tag{39}$$

Now above is strictly true for linear equations but approximately true for non-linear equations where guessed initial values are near the true values.

In above method of interpolation for obtaining correct initial values, effect of modifications of all initial values on any particular final value has been taken into consideration. Thus in non-linear case it can be hoped that convergence to the correct initial value will be obtained more rapidly in the scheme of interpolation than that can be obtained in a scheme where interpolation is done only between one initial value and corresponding final value.

5.2 Example

General program can best be understood with reference to some equations to be solved. Let us take the equations (15) and (16) as an example.

Consider the following equation

$$f''' + B\,(ff'' + (s - f'^2)\,\beta) + f''\,(1-B) = 0$$
$$s'' + B\,fs'\,Pr + s' - Bs' = 0 \qquad (40)$$

with boundary conditions $f'(0) = f(0) = 0,\ f'(\infty) = 1$

$$s\,(0) = Sw,\ s(\infty) = 1$$

Above set of equations is reducible to equations (15, 16) for B ≠equal to unity. Now for B = 0, equation (40) is reduced to a very simple form (41)

$$f''' + f''(1-B) = 0$$
$$s'' + s' = 0 \qquad (41)$$

whose solution is given by

$$f = \eta + e^{-\eta} - 1$$
$$s = 1 + (Sw - 1)\,e^{-\eta} \qquad (41a)$$

From the solution of equations (40) for B equal to ~~0~~zero, we can march forward by parametric differentiation method to get the solution at B equal to unity.

Though equation (40) looks more complicated than the parent equation nonetheless it is chosen as it gives the starting solution readily.

Now differentiating (40) with respect to B we get

$$G''' + (ff' + (s - f'^2) + B(Gf'' + fG'' + (T - 2f')\beta) - f'' - G''(1 - B) = 0$$
$$T'' + fs'\,Pr + B\,Gs'\,Pr + BfT'Pr + T' - BT' - s' = 0 \qquad (42)$$

with boundary conditions

$$G(0) = G'(0) = G'(\infty) = 0$$

and $\quad T(0) = T(\infty) = 0$

where

$$G = \frac{df}{dB}, \quad T = \frac{ds}{dB}$$

Parent equations are

$$f''' + ff'' + (s - f'^2)\beta = 0$$
$$s'' + fs'\,Pr = 0 \qquad (43)$$

with boundary conditions

$$f(0) = f'(0) = 0$$
$$f'(\infty) = 1$$

and $\quad s(0) = Sw, \; s(\infty) = 1$

Now we have 3 sets of equations. Ist set is equation (41), second set is (42) and third one is eqn. (43). In PDM (Parametric Differentiation Method), we first solve Ist set, then second set. If higher accuracy is required we solve the third set of equations directly. In solving the third set we obtained starting values by solving the second set of equations by PDM.

In this program any dependent variable is represented as double subscripted variable Q(n, m) where n denotes value of the independent variable at which dependent variable Q is beihg considered, and m denotes different dependent variables. Correspondence between m and different dependent variable is done in a

systematic way. With the order in which equations are written in (42) program assigns

$$G = Q(n, 1)$$
$$G' = Q(n, 2)$$
$$G'' = Q(n, 3)$$
$$G''' = Q(n, 4)$$

and

$$T = Q(n, 5)$$
$$T' = Q(n, 6)$$
$$T'' = Q(n, 7)$$

Such correspondence in general program is obtained by introducing some artificial integer variables whose values can be fed in the program as data. Values of these variables are different for different set of equations.

The artificial variable are JJEQ, JJCI, JJCF, NKMP, LCH, NIB, NFB, MI, NNV.

In addition to these variables there are others like SST, FFN, NR, H, DB, JA.

Some of these variables are double subscripted, right hand side subscript showing the set of equations to which these variables belong.

Now a short description of these variables are given below.

JJEQ is a double subscripted variable which is given value 0 or 1. If

$$JJEQ\ (n, m) = 1$$

it denotes highest derivative of dependent variable occurring in one of the equations of m^{th} set is the n^{th} dependent variable in the program.

Hence corresponding to equations(42)

JJEQ (4,2) = 1, JJEQ (7, 2) = 1

JJEQ (4, 2) denotes occurance of G''' and JJEQ(7, 2) denotes that of T'' in the second set of equations.

JJCI is a double subscripted variable which tells the program what the missing starting values are. Like JJEQ, JJCI is given values 0 or 1. Now if

JJCI (n, m) = 1

it denotes initial value of the n^{th} variable is missing in m^{th} set of equations. Hence, for equation (42), we have

JJCI (3,2) = 1
JJCI (6,2) = 1

These correspond to G'' and T' whose initial values are not know

JJCF is also a double subscripted variable denoting the variables which have prescribed values at the end point. If JJCF (n, m) = 1 it denotes n^{th} variable of the m^{th} set of equation, has a prescribed value at the end point. For eq. (4 we have

JJCF (2,2) = 1
JJCF (5,2) = 1

These variables take the value 1 under the conditions mentione above, otherwise they take the value zero.

LCH is a subscripted variables. It can take the value 0, 1, 2, 3. If LCH (m) equal to zero, m^{th} set of equation will be calculated as will be explained while describing subroutine B

If LCH (m) equal to 1, the m^{th} set will be calculated by PDM. If LCH(m) equal to 2, the m^{th} set will be solved directly after obtaining approximate starting values by PDM. If LCH (m) equal to 3, m^{th} set will be solved directly and values obtained in solving the previous set of equations will also be used in solving this (m^{th}) set.

NIB (m) and NFB (m) gives respectively the number of initial and final boundary condition given for the m^{th} set of equation. Here, their values are 3 and 2 respectively for all sets of equations.

NKMP is the total number of sets of equations to be solved. Hence here NKMP is equal to 3 (as three sets of equations are to be solved here).

With these artificial variables, program reads the identity of the equation. Other variables like H,NNV,NR,SST,FFN supply the actual data like range of independent variable, values given at the boundary points, etc.

A short description of these variables are given below

NNV(m) denotes the number of dependent variable as will be interpretated by the program in the m^{th} set of equation, i.e. it is equal to the sum of the maximum order of derivatives of different dependent variable occurring in the m^{th} set of equations and number of actual dependent variable in the m^{th} set of equation.

Here for the second set of equations

maximum order of the derivative of G = 3

maximum order of the derivative of T= 2

no. of dependent variables(G and T) is 2.

Hence

NNV (2) = 3 + 2 + 2 = 7.

NR = No. of steps into which domain of independent variable is divided.

H = step size of the independent variable.

SST and FFN are double subscripted variables corresponding to the boundary conditions given at initial and end points. Here G(0), G'(0), T(0) are given for the second set of equations we assigns these values to SST(1, 2) SST(2, 2), SST(3, 2) respectively. Similarly the end point G" (∞) and T (∞) are given, hence we assign these values to FFN(1, 2) ahd FFN (2, 2) respectively.

First initial value corresponding to the order in which different dependent (variables are arranged (as interpreted by program) is assigned to SST (2 , 2), the second to SST (2, 2) and so on. The order in which the dependent variable is arranged

G, G', G'', G'''. T, T', T''

Now first initial condition corresponds to G, second to G', third to T. Hence

SST(1, 2) = G(0) = 0
SST(2, 2) = G'(0)= 0
SST(3, 2) = T'(0)= 0

Similarly,

$$FFN(1) = G''(\infty) = 0$$

$$FFN(2) = T(\infty) = 0$$

Variable MI controls the accuracy to which boundary conditions to be satisfied. If MI is given a value 3, minimum upto third place of decimal, boundary condition will be satisfied.

5.3 Subroutines

There are eight subroutines in this program. They are as given below.

MIST -This subroutine along with subroutines RKM solves the equation by PDM. This is effected for the second set of equatio by putting LCH(2) = 1

ANT - This subroutine using the values of JJEQ, JJCI, JJCF, etc. defines some other variables like JQ, JC, Ietc. Values assigned to these variables are used in the main program and subroutine RKM.

HIX - This subroutine along with RKM, solves the original equation directly. Here original equation is the third set. Hence, we equate LCH (3) to two. Due to this value assigned to LCH for the third set this set will be solved directly utilizing starting initial condition as obtained solving second set by PDM.

RKM - This is the most important subroutine which solves a particular set of equations depending on the value attained by KMP in the main program. KMP denotes the number of the set to be solved When in the main program KMP attains the value 2,

second set of equation is solved by the program. As mentioned earlier, this subroutine iterates the result to the desired accuracy controlled by the value assigned to MI .

ALEQ - This is a subroutine which solves linear algebraic equatior This subroutine is called from RKM for getting new interpolated initial values.

TICK - This is a subroutine which calculate derivatives one order higher. In RKM, this is called to obtain higher derivative from lower ones.

FUNSON - This is the subroutine where different sets of equation are fed in the following way. Let us consider second set of equa- tions which can be written as

$$G''' = -\left(ff' + (s - f'^2)\beta + B(gf'' + fG'' + (T - 2f'G')\beta) - f'' + G''(1-B) \right)$$

$$T'' = -\left(fs'\,Pr + B\,G\,s'\,Pr + BfT'\,Pr + T' - BT' - s' \right)$$

Now this can be written in terms of subscripted variables as

FOMULA = - (F(1)*F(2)+(F(5)-F(2)*F(2))*β+B*(Q(1)*F(3)+F(1)*

+ (Q(5) - 2*F(2)*Q(2)) * β - F(3)+Q(3)(1-B))

where G''' is replaced by a variable FOMULA

variable F corresponds to f and s and Q corresponds to G and T. subscript of F and Q are decided by the same rule as mentioned in case of subscript m of double subscripted variable F(n, m). While writing down equations in FUNSON subroutine we should remember variable like G, T, G' etc. which are actually being evaluated in RKM by Runge-Kutta Method is denoted by Q and other variables like f, f', s etc. which has already been calculated and is being

utilised for calculating G, T, G' etc. or Q are denoted by F. The statement bearing formula given above is assigned a statement number given by

N = KMP*100+M

where M is equal to the position of a equation in the KMPth set of equations. The above equation is the first equation of second set. Hence,

N = 2*100+1 = 201

Hence, equation (44) is transferred to FUNSON subroutine as (for β = 1).

```
201 FOMULA   = - (F(1)*F(2)+F(5)-F(2)*F(2))+B*(G(1)*F(3)+F(1)*Q(3)
               + (Q(5)-2*F(2)*Q(2))*1-F(3)+Q(3)*(1-B))
```

Similarly, equation (45) is transferred as (for Pr = 1)

```
202 FOMULA   = - (F(1)*F(6)+B*Q(1)*F(6)+B*F(1)*Q(6)+Q(6)
               -B*Q(6) - F(6))
```

When the third set is to be solved, corresponding equations are transferred to the FUNSON subroutine as (for β = 1, Pr = 1)

```
301 FOMULA   =  - (Q(1)*Q(3)+Q(2)**2))
302 FOMULA   =  - Q(1)*Q(6)
```

BST - If solution for the initial value of the parameter can be obtained directly as in the case of equation (41) where solution is obtainable in a closed form as given by (41a), then the formula giving the solution for the initial value of parameter is fed in

the subroutine BST. This is achieved by giving the value equal to zero to the variable LCH for the set of the equation for which the solution is obtainable in a closed form. Here equation (41) is the first set. Hence we put

LCH (1) = 0.

5.4 Some comments on the General Program:

To use the general program, most economically and effectively it is suggested that first the problem should be solved by PDM taking large step size of the parameter. After obtaining the solution at the desired value of the parameter, we can solve the original non-linear equations directly with starting values obtained by PDM in the first step. As the assumed initial values are very near the exact value, solution will converge to the true solution of the desired accuracy in a few iterations. Solution obtained by PDM by taking a step size of β equal to .25 and the solution obtained by solving the parent equations directly utilizing the initial boundary values as obtained by PDM, are given in a tabular form on the next page.

In the main program, double subscript variable AF always denotes the variable whose values are to evaluated like (f, f', s, s' etc. The other double subscripted variable is Q which is the variable whose values are known. In case of PDM, Q denotes variables like (df/dβ), (df'/dB) etc.

Table 1(a)

ETA(η)	f	s
0.0	0.000	1.5000
0.5	0.1552	1.352
1.0	0.5171	1.217
1.5	0.9722	1.113
2.0	1.460	1.048
2.5	1.956	1.017
3.0	2.453	1.003
3.5	2.950	1.001
4.0	3.448	1.001
4.5	3.946	1.001
5.0	4.445	1.001
5.5	4.944	1.000
6.0	5.443	1.000
6.5	5.942	1.000
7.0	6.442	1.000
7.5	6.942	1.000

T Solution obtained by solving equation (42) by PDM taking B = 0.25 for $\beta=0, Pr=1$

Table 1(b)

ETA(η)	f	s
0.0	0.000	1.500
0.5	0.1565	1.352
1.0	0.5213	1.217
1.5	0.9800	1.112
2.0	1.472	1.048
2.5	1.971	1.017
3.0	2.472	1.005
3.5	2.972	1.001
4.0	3.472	1.000
4.5	3.972	1.000
5.0	4.472	1.000
5.5	4.972	1.000
6.0	5.472	1.000
6.5	5.972	1.000
7.0	6.472	1.000
7.5	6.972	1.000

Solution obtained by directly solving the parent nonlinear equations starting value of which is obtained by solving equation (42) by PDM. Here also $\beta=0, Pr=1$.

The program can also be utilised to solve more than one set of equations where results of one set of equations is to be used in the solution of the next set as is required in solving second order boundary layer equations where values from the first order is required to solve the second order equations. In this case Q can be given values corresponding to the first order solutions. This is achieved in the program by giving the value 3 to the variable LCH for the set of equation corresponding to second order boundary equations.

General program is given in the Appendix 3. This program can solve a set of equations containing 25 (computer) dependent variables. On IBM 7044 computer, some 4800 memory cores remain unused. Hence by changing dimension, this program can handle equations with 35 dependent variable. Maximum number of boundary condition that can be satisfied at the end point is 10. Time taken by this program on IBM 7044 to solve equation (42) by PDM and then to solve equation (43) directly is 6 minutes 53 seconds.

DISCUSSION

Results obtained for Prandtl number equal to unity tally quite fairly with those given by Cohen and Reshotko (Ref. 4). General trend of the result is the same as given in that reference (4). Maximum deviation of our result from that given in that reference is .5 percent. This difference can still be reduced by taking shorter step size of parameter β and that of the independent variable . From the result obtained for Prandtl number eual to unity, we can hope results obtained for other Prandtl number are also reliable so far as the solution of equation (15) and (16) is concerned.

For Prandtl number not equal to unity, solution obtained, in general is the complementary function of equation (13) but gives the near exact solution for low mach number flows. For high mach number ($M \to \infty$) right hand side of eqnation (13) approaches a constant value equal to two. With slight modification in programme results can be obtained for high mach number. Thus solution obtained for Prandtl number not equal to one may not give exact quantitative value in all cases. Nonetheless, values were calculated for Prandtl number not equal to unity as it gives a qualitative understanding of boundary layer.

Our main aim was to solve the compressible boundary layer equation by parametric differentiation method and we show that this method can be effectively applied to solve the problem.

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A P P E N D I X 1

TABLE I

BETA = 0.0000   PRANDTLE NO. = 1.0000   SW = 0.0000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = 0.4695

DISPLACEMENT THICKNESS = -0.0001

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = -1.2169

REYNOLD ANALOGY PARAMETER = 2.0004

ETA	F	DF	S
0.	0.	0.1428E-07	0.
0.5000E 00	0.5864E-01	0.2342E 00	0.2342E 00
0.1000E 01	0.2330E 00	0.4606E 00	0.4606E 00
0.1500E 01	0.5150E 00	0.6615E 00	0.6614E 00
0.2000E 01	0.8868E 00	0.8167E 00	0.8167E 00
0.2500E 01	0.1322E 01	0.9168E 00	0.9168E 00
0.3000E 01	0.1796E 01	0.9691E 00	0.9690E 00
0.3500E 01	0.2286E 01	0.9907E 00	0.9907E 00
0.4000E 01	0.2784E 01	0.9978E 00	0.9978E 00
0.4500E 01	0.3283E 01	0.9996E 00	0.9996E 00
0.5000E 01	0.3783E 01	0.9999E 00	0.9999E 00
0.5500E 01	0.4283E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4783E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5283E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5783E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6283E 01	0.1000E 01	0.1000E 01

TABLE 2

BETA = 1.0000 PRANDTLE NO. = 1.0000 SW = 0.0000

VELOCITY GRADIENT AT WALL = 0.6486

TEMPERATURE GRADIENT AT WALL = 0.5065

DISPLACEMENT THICKNESS = -0.1575

MOMENTUM THICKNESS = 0.4036

THERMAL THICKNESS = -1.1374

REYNOLD ANALOGY PARAMETER = 2.5612

ETA	F	DF	S
0.	0.	0.3249E-07	0.
0.5000E 00	0.7987E-01	0.3149E 00	0.2524E 00
0.1000E 01	0.3067E 00	0.5817E 00	0.4936E 00
0.1500E 01	0.6494E 00	0.7762E 00	0.6998E 00
0.2000E 01	0.1071E 01	0.8971E 00	0.8492E 00
0.2500E 01	0.1537E 01	0.9602E 00	0.9374E 00
0.3000E 01	0.2025E 01	0.9873E 00	0.9789E 00
0.3500E 01	0.2521E 01	0.9967E 00	0.9943E 00
0.4000E 01	0.3020E 01	0.9993E 00	0.9988E 00
0.4500E 01	0.3520E 01	0.9999E 00	0.9998E 00
0.5000E 01	0.4020E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.4520E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5020E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5520E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6020E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6520E 01	0.1000E 01	0.1000E 01

TABLE 3

BETA = 2.0000 PRANDTLE NO. = 1.0000 SW = 0.0000

VELOCITY GRADIENT AT WALL = 0.7384

TEMPERATURE GRADIENT AT WALL = 0.5205

DISPLACEMENT THICKNESS = -0.2060

MOMENTUM THICKNESS = 0.3839

THERMAL THICKNESS = -1.1109

REYNOLD ANALOGY PARAMETER = 2.8374

ETA	F	DF	S
0.	0.	-0.3332E-07	0.
0.5000E 00	0.9001E-01	0.3517E 00	0.2593E 00
0.1000E 01	0.3385E 00	0.6270E 00	0.5056E 00
0.1500E 01	0.7014E 00	0.8099E 00	0.7131E 00
0.2000E 01	0.1136E 01	0.9156E 00	0.8596E 00
0.2500E 01	0.1608E 01	0.9682E 00	0.9434E 00
0.3000E 01	0.2099E 01	0.9900E 00	0.9815E 00
0.3500E 01	0.2596E 01	0.9975E 00	0.9952E 00
0.4000E 01	0.3095E 01	0.9995E 00	0.9990E 00
0.4500E 01	0.3595E 01	0.9999E 00	0.9998E 00
0.5000E 01	0.4095E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.4595E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5095E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5595E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6095E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6595E 01	0.1000E 01	0.1000E 01

TABLE 4

BETA = 0.0000 PRANDTLE NO. = 1.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = 0.0939

DISPLACEMENT THICKNESS = 0.9734

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = -0.2434

REYNOLD ANALOGY PARAMETER = 2.0004

ETA	F	DF	S
0.	0.	0.1428E-07	0.8000E 00
0.5000E 00	0.5864E-01	0.2342E 00	0.8468E 00
0.1000E 01	0.2330E 00	0.4606E 00	0.8921E 00
0.1500E 01	0.5150E 00	0.6615E 00	0.9323E 00
0.2000E 01	0.8868E 00	0.8167E 00	0.9633E 00
0.2500E 01	0.1322E 01	0.9168E 00	0.9834E 00
0.3000E 01	0.1796E 01	0.9691E 00	0.9938E 00
0.3500E 01	0.2286E 01	0.9907E 00	0.9981E 00
0.4000E 01	0.2784E 01	0.9978E 00	0.9996E 00
0.4500E 01	0.3283E 01	0.9996E 00	0.9999E 00
0.5000E 01	0.3783E 01	0.9999E 00	0.1000E 01
0.5500E 01	0.4283E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4783E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5283E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5783E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6283E 01	0.1000E 01	0.1000E 01

TABLE 5

BETA = 1.0000 PRANDTLE NO. = 1.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 1.1234

TEMPERATURE GRADIENT AT WALL = 0.1119

DISPLACEMENT THICKNESS = 0.4950

MOMENTUM THICKNESS = 0.3163

THERMAL THICKNESS = -0.2088

REYNOLD ANALOGY PARAMETER = 4.0165

ETA	F	DF	S
0.	0.	-0.9810E-07	0.8000E 00
0.5000E 00	0.1238E 00	0.4622E 00	0.8556E 00
0.1000E 01	0.4320E 00	0.7442E 00	0.9076E 00
0.1500E 01	0.8458E 00	0.8934E 00	0.9494E 00
0.2000E 01	0.1312E 01	0.9616E 00	0.9770E 00
0.2500E 01	0.1801E 01	0.9881E 00	0.9914E 00
0.3000E 01	0.2297E 01	0.9969E 00	0.9974E 00
0.3500E 01	0.2796E 01	0.9993E 00	0.9994E 00
0.4000E 01	0.3296E 01	0.9999E 00	0.9999E 00
0.4500E 01	0.3796E 01	0.1000E 01	0.1000E 01
0.5000E 01	0.4296E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.4796E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5296E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5796E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6296E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6796E 01	0.1000E 01	0.1000E 01

TABLE 6

BETA = 2.0000 PRANDTLE NO. = 1.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 1.5127

TEMPERATURE GRADIENT AT WALL = 0.1182

DISPLACEMENT THICKNESS = 0.3634

MOMENTUM THICKNESS = 0.2637

THERMAL THICKNESS = -0.1993

REYNOLD ANALOGY PARAMETER = 5.1173

ETA	F	DF	S
0.	0.	-0.1582E-05	0.8000E 00
0.5000E 00	0.1566E 00	0.5648E 00	0.8587E 00
0.1000E 01	0.5150E 00	0.8321E 00	0.9126E 00
0.1500E 01	0.9626E 00	0.9410E 00	0.9543E 00
0.2000E 01	0.1445E 01	0.9810E 00	0.9803E 00
0.2500E 01	0.1939E 01	0.9945E 00	0.9931E 00
0.3000E 01	0.2438E 01	0.9986E 00	0.9980E 00
0.3500E 01	0.2937E 01	0.9997E 00	0.9996E 00
0.4000E 01	0.3437E 01	0.9999E 00	0.9999E 00
0.4500E 01	0.3937E 01	0.1000E 01	0.1000E 01
0.5000E 01	0.4437E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.4937E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5437E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5937E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6437E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6937E 01	0.1000E 01	0.1000E 01

TABLE 7

BETA = 0.0000 PRANDTLE NO. = 1.0000 SW = 1.0000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = 0.0000

DISPLACEMENT THICKNESS = 1.2168

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = -0.0000

ETA	F	DF	S
0.	0.	0.1428E-07	0.1000E 01
0.5000E 00	0.5864E-01	0.2342E 00	0.1000E 01
0.1000E 01	0.2330E 00	0.4606E 00	0.1000E 01
0.1500E 01	0.5150E 00	0.6615E 00	0.1000E 01
0.2000E 01	0.8868E 00	0.8167E 00	0.1000E 01
0.2500E 01	0.1322E 01	0.9168E 00	0.1000E 01
0.3000E 01	0.1796E 01	0.9691E 00	0.1000E 01
0.3500E 01	0.2286E 01	0.9907E 00	0.1000E 01
0.4000E 01	0.2784E 01	0.9978E 00	0.1000E 01
0.4500E 01	0.3283E 01	0.9996E 00	0.1000E 01
0.5000E 01	0.3783E 01	0.9999E 00	0.1000E 01
0.5500E 01	0.4283E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4783E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5283E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5783E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6283E 01	0.1000E 01	0.1000E 01

TABLE B

BETA = 1.0000 PRANDTLE NO. = 1.0000 SW = 1.0000

VELCCITY GRADIENT AT WALL = 1.2317

TEMPERATURE GRADIENT AT WALL = -0.0000

DISPLACEMENT THICKNESS = 0.6497

MOMENTUM THICKNESS = 0.2937

THERMAL THICKNESS = -0.0000

ETA	F	DF	S
0.	0.	-0.1525E-06	0.1000E 01
0.5000E 00	0.1335E 00	0.4943E 00	0.1000E 01
0.1000E 01	0.4598E 00	0.7772E 00	0.1000E 01
0.1500E 01	0.8866E 00	0.9153E 00	0.1000E 01
0.2000E 01	0.1361E 01	0.9725E 00	0.1000E 01
0.2500E 01	0.1853E 01	0.9923E 00	0.1000E 01
0.3000E 01	0.2351E 01	0.9981E 00	0.1000E 01
0.3500E 01	0.2850E 01	0.9996E 00	0.1000E 01
0.4000E 01	0.3350E 01	0.9999E 00	0.1000E 01
0.4500E 01	0.3850E 01	0.1000E 01	0.1000E 01
0.5000E 01	0.4350E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.4850E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5350E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5850E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6350E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6850E 01	0.1000E 01	0.1000E 01

TABLE 9

BETA = 0.0000 PRANDTLE NO. = 1.0000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = -0.2348

DISPLACEMENT THICKNESS = 1.8252

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = 0.6084

ETA	F	DF	S
0.	0.	0.1428E-07	0.1500E 01
0.5000E 00	0.5864E-01	0.2342E 00	0.1383E 01
0.1000E 01	0.2330E 00	0.4606E 00	0.1270E 01
0.1500E 01	0.5150E 00	0.6615E 00	0.1169E 01
0.2000E 01	0.8868E 00	0.8167E 00	0.1092E 01
0.2500E 01	0.1322E 01	0.9168E 00	0.1042E 01
0.3000E 01	0.1796E 01	0.9691E 00	0.1015E 01
0.3500E 01	0.2286E 01	0.9907E 00	0.1005E 01
0.4000E 01	0.2784E 01	0.9978E 00	0.1001E 01
0.4500E 01	0.3283E 01	0.9996E 00	0.1000E 01
0.5000E 01	0.3783E 01	0.9999E 00	0.1000E 01
0.5500E 01	0.4283E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4783E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5283E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5783E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6283E 01	0.1000E 01	0.1000E 01

TABLE 10

BETA = 1.0000 PRANDTLE NO. = 1.0000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 1.4904

TEMPERATURE GRADIENT AT WALL = -0.2970

DISPLACEMENT THICKNESS = 1.0260

MOMENTUM THICKNESS = 0.2366

THERMAL THICKNESS = 0.4954

ETA	F	DF	S
0.	0.	-0.3003E-06	0.1500E 01
0.5000E 00	0.1563E 00	0.5686E 00	0.1353E 01
0.1000E 01	0.5208E 00	0.8511E 00	0.1217E 01
0.1500E 01	0.9790E 00	0.9621E 00	0.1113E 01
0.2000E 01	0.1470E 01	0.9944E 00	0.1048E 01
0.2500E 01	0.1969E 01	0.1000E 01	0.1017E 01
0.3000E 01	0.2469E 01	0.1000E 01	0.1005E 01
0.3500E 01	0.2969E 01	0.1000E 01	0.1001E 01
0.4000E 01	0.3469E 01	0.1000E 01	0.1000E 01
0.4500E 01	0.3969E 01	0.1000E 01	0.1000E 01
0.5000E 01	0.4469E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.4969E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5469E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5969E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6469E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6969E 01	0.1000E 01	0.1000E 01

TABLE 11

BETA = 0.0000 PRANDTLE NO. = 0.3000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = 0.0608

DISPLACEMENT THICKNESS = 0.8356

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = -0.3811

REYNOLD ANALOGY PARAMETER = 3.0915

ETA	F	DF	S
0.	0.	0.1428E-07	0.8000E 00
0.5000E 00	0.5864E-01	0.2342E 00	0.8304E 00
0.1000E 01	0.2330E 00	0.4606E 00	0.8604E 00
0.1500E 01	0.5150E 00	0.6615E 00	0.8894E 00
0.2000E 01	0.8868E 00	0.8167E 00	0.9162E 00
0.2500E 01	0.1322E 01	0.9168E 00	0.9397E 00
0.3000E 01	0.1796E 01	0.9691E 00	0.9589E 00
0.3500E 01	0.2286E 01	0.9907E 00	0.9737E 00
0.4000E 01	0.2784E 01	0.9978E 00	0.9841E 00
0.4500E 01	0.3283E 01	0.9996E 00	0.9910E 00
0.5000E 01	0.3783E 01	0.9999E 00	0.9953E 00
0.5500E 01	0.4283E 01	0.1000E 01	0.9977E 00
0.6000E 01	0.4783E 01	0.1000E 01	0.9990E 00
0.6500E 01	0.5283E 01	0.1000E 01	0.9996E 00
0.7000E 01	0.5783E 01	0.1000E 01	0.9999E 00
0.7500E 01	0.6283E 01	0.1000E 01	0.1000E 01

TABLE 12

BETA = 0.5000 PRANDTLE NO. = 0.3000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.8448

TEMPERATURE GRADIENT AT WALL = 0.0662

DISPLACEMENT THICKNESS = 0.5298

MOMENTUM THICKNESS = 0.3885

THERMAL THICKNESS = -0.3558

REYNOLD ANALOGY PARAMETER = 5.1038

ETA	F	DF	S
0.	0.	0.7792E-07	0.8000E 00
0.5000E 00	0.9719E-01	0.3718E 00	0.8331E 00
0.1000E 01	0.3548E 00	0.6420E 00	0.8656E 00
0.1500E 01	0.7233E 00	0.8176E 00	0.8964E 00
0.2000E 01	0.1160E 01	0.9172E 00	0.9240E 00
0.2500E 01	0.1632E 01	0.9661E 00	0.9472E 00
0.3000E 01	0.2121E 01	0.9870E 00	0.9654E 00
0.3500E 01	0.2617E 01	0.9951E 00	0.9787E 00
0.4000E 01	0.3115E 01	0.9981E 00	0.9877E 00
0.4500E 01	0.3615E 01	0.9992E 00	0.9933E 00
0.5000E 01	0.4115E 01	0.9997E 00	0.9966E 00
0.5500E 01	0.4614E 01	0.9999E 00	0.9984E 00
0.6000E 01	0.5114E 01	0.1000E 01	0.9993E 00
0.6500E 01	0.5614E 01	0.1000E 01	0.9997E 00
0.7000E 01	0.6114E 01	0.1000E 01	0.9999E 00
0.7500E 01	0.6614E 01	0.1000E 01	0.1000E 01

TABLE 13

BETA = 1.0000 PRANDTLE NO. = 0.3000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 1.1003

TEMPERATURE GRADIENT AT WALL = 0.0687

DISPLACEMENT THICKNESS = 0.4058

MOMENTUM THICKNESS = 0.3490

THERMAL THICKNESS = -0.3456

REYNOLD ANALOGY PARAMETER = 6.4044

ETA	F	DF	S
0.	0.	-0.9696E-07	0.8000E 00
0.5000E 00	0.1210E 00	0.4515E 00	0.8343E 00
0.1000E 01	0.4219E 00	0.7263E 00	0.8679E 00
0.1500E 01	0.8261E 00	0.8737E 00	0.8994E 00
0.2000E 01	0.1283E 01	0.9443E 00	0.9273E 00
0.2500E 01	0.1764E 01	0.9755E 00	0.9503E 00
0.3000E 01	0.2255E 01	0.9890E 00	0.9680E 00
0.3500E 01	0.2752E 01	0.9949E 00	0.9806E 00
0.4000E 01	0.3250E 01	0.9977E 00	0.9890E 00
0.4500E 01	0.3749E 01	0.9990E 00	0.9941E 00
0.5000E 01	0.4249E 01	0.9996E 00	0.9971E 00
0.5500E 01	0.4749E 01	0.9998E 00	0.9987E 00
0.6000E 01	0.5249E 01	0.9999E 00	0.9994E 00
0.6500E 01	0.5749E 01	0.1000E 01	0.9998E 00
0.7000E 01	0.6249E 01	0.1000E 01	0.9999E 00
0.7500E 01	0.6749E 01	0.1000E 01	0.1000E 01

TABLE 14

BETA = 1.5000 PRANDTLE NO. = 0.3000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 1.3064

TEMPERATURE GRADIENT AT WALL = 0.0703

DISPLACEMENT THICKNESS = 0.3331

MOMENTUM THICKNESS = 0.3243

THERMAL THICKNESS = -0.3396

REYNOLD ANALOGY PARAMETER = 7.4323

ETA	F	DF	S
0.	0.	-0.6205E-06	0.8000E 00
0.5000E 00	0.1388E 00	0.5081E 00	0.8351E 00
0.1000E 01	0.4680E 00	0.7771E 00	0.8694E 00
0.1500E 01	0.8919E 00	0.9018E 00	0.9013E 00
0.2000E 01	0.1358E 01	0.9554E 00	0.9293E 00
0.2500E 01	0.1842E 01	0.9785E 00	0.9521E 00
0.3000E 01	0.2335E 01	0.9892E 00	0.9694E 00
0.3500E 01	0.2831E 01	0.9946E 00	0.9817E 00
0.4000E 01	0.3329E 01	0.9974E 00	0.9897E 00
0.4500E 01	0.3828E 01	0.9988E 00	0.9946E 00
0.5000E 01	0.4328E 01	0.9995E 00	0.9973E 00
0.5500E 01	0.4827E 01	0.9998E 00	0.9988E 00
0.6000E 01	0.5327E 01	0.9999E 00	0.9995E 00
0.6500E 01	0.5827E 01	0.1000E 01	0.9998E 00
0.7000E 01	0.6327E 01	0.1000E 01	0.9999E 00
0.7500E 01	0.6827E 01	0.1000E 01	0.1000E 01

TABLE 15

BETA = 2.0000 PRANDTLE NO. = 0.3000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 1.4837

TEMPERATURE GRADIENT AT WALL = 0.0714

DISPLACEMENT THICKNESS = 0.2836

MOMENTUM THICKNESS = 0.3069

THERMAL THICKNESS = -0.3355

REYNOLD ANALOGY PARAMETER = 8.3069

ETA	F	DF	S
0.	0.	-0.1548E-05	0.8000E 00
0.5000E 00	0.1531E 00	0.5517E 00	0.8356E 00
0.1000E 01	0.5028E 00	0.8111E 00	0.8704E 00
0.1500E 01	0.9391E 00	0.9180E 00	0.9026E 00
0.2000E 01	0.1410E 01	0.9609E 00	0.9306E 00
0.2500E 01	0.1896E 01	0.9796E 00	0.9533E 00
0.3000E 01	0.2389E 01	0.9891E 00	0.9704E 00
0.3500E 01	0.2885E 01	0.9943E 00	0.9824E 00
0.4000E 01	0.3383E 01	0.9972E 00	0.9901E 00
0.4500E 01	0.3882E 01	0.9987E 00	0.9949E 00
0.5000E 01	0.4381E 01	0.9994E 00	0.9975E 00
0.5500E 01	0.4881E 01	0.9998E 00	0.9989E 00
0.6000E 01	0.5381E 01	0.9999E 00	0.9995E 00
0.6500E 01	0.5881E 01	0.1000E 01	0.9998E 00
0.7000E 01	0.6381E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6881E 01	0.1000E 01	0.1000E 01

TABLE 16

BETA = 0.0000 PRANDTLE NO. = 0.3000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = -0.1519

DISPLACEMENT THICKNESS = 2.1696

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = 0.9528

REYNOLD ANALOGY PARAMETER = 3.0915

ETA	F	DF	S
0.	0.	0.1428E-07	0.1500E 01
0.5000E 00	0.5664E-01	0.2342E 00	0.1424E 01
0.1000E 01	0.2330E 00	0.4606E 00	0.1349E 01
0.1500E 01	0.5150E 00	0.6615E 00	0.1277E 01
0.2000E 01	0.8868E 00	0.8167E 00	0.1210E 01
0.2500E 01	0.1322E 01	0.9168E 00	0.1151E 01
0.3000E 01	0.1796E 01	0.9691E 00	0.1103E 01
0.3500E 01	0.2286E 01	0.9907E 00	0.1066E 01
0.4000E 01	0.2784E 01	0.9978E 00	0.1040E 01
0.4500E 01	0.3283E 01	0.9996E 00	0.1022E 01
0.5000E 01	0.3783E 01	0.9999E 00	0.1012E 01
0.5500E 01	0.4283E 01	0.1000E 01	0.1006E 01
0.6000E 01	0.4783E 01	0.1000E 01	0.1003E 01
0.6500E 01	0.5283E 01	0.1000E 01	0.1001E 01
0.7000E 01	0.5783E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6283E 01	0.1000E 01	0.1000E 01

TABLE 17

BETA = 1.0000 PRANDTLE NO. = 0.3000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 1.5430

TEMPERATURE GRADIENT AT WALL = -0.1842

DISPLACEMENT THICKNESS = 1.2457

MOMENTUM THICKNESS = 0.1543

THERMAL THICKNESS = 0.8146

REYNOLD ANALOGY PARAMETER = 8.3772

ETA	F	DF	S
0.	0.	-0.2964E-06	0.1500E 01
0.5000E 00	0.1626E 00	0.5930E 00	0.1408E 01
0.1000E 01	0.5435E 00	0.8905E 00	0.1319E 01
0.1500E 01	0.1023E 01	0.1005E 01	0.1236E 01
0.2000E 01	0.1534E 01	0.1030E 01	0.1166E 01
0.2500E 01	0.2048E 01	0.1025E 01	0.1109E 01
0.3000E 01	0.2558E 01	0.1015E 01	0.1068E 01
0.3500E 01	0.3064E 01	0.1008E 01	0.1040E 01
0.4000E 01	0.3567E 01	0.1004E 01	0.1022E 01
0.4500E 01	0.4068E 01	0.1002E 01	0.1011E 01
0.5000E 01	0.4569E 01	0.1001E 01	0.1005E 01
0.5500E 01	0.5069E 01	0.1000E 01	0.1002E 01
0.6000E 01	0.5569E 01	0.1000E 01	0.1001E 01
0.6500E 01	0.6069E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6569E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.7069E 01	0.1000E 01	0.1000E 01

TABLE 18

BETA = 2.0000 PRANDTLE NO. = 0.3000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 2.1637

TEMPERATURE GRADIENT AT WALL = -0.1936

DISPLACEMENT THICKNESS = 1.0260

MOMENTUM THICKNESS = 0.0417

THERMAL THICKNESS = 0.7831

REYNOLD ANALOGY PARAMETER = 11.1743

ETA	F	DF	S
0.	0.	-0.3973E-05	0.1500E 01
0.5000E 00	0.2117E 00	0.7401E 00	0.1403E 01
0.1000E 01	0.6606E 00	0.1003E 01	0.1310E 01
0.1500E 01	0.1131E 01	0.1059E 01	0.1226E 01
0.2000E 01	0.1710E 01	0.1052E 01	0.1155E 01
0.2500E 01	0.2231E 01	0.1034E 01	0.1100E 01
0.3000E 01	0.2744E 01	0.1019E 01	0.1061E 01
0.3500E 01	0.3251E 01	0.1010E 01	0.1035E 01
0.4000E 01	0.3754E 01	0.1005E 01	0.1018E 01
0.4500E 01	0.4256E 01	0.1002E 01	0.1009E 01
0.5000E 01	0.4757E 01	0.1001E 01	0.1004E 01
0.5500E 01	0.5257E 01	0.1000E 01	0.1002E 01
0.6000E 01	0.5757E 01	0.1000E 01	0.1001E 01
0.6500E 01	0.6257E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6757E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.7257E 01	0.1000E 01	0.1000E 01

BETA = 0.0000 PRANDTLE NO. = 5.0000 SW = 1.0000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = 0.0000

DISPLACEMENT THICKNESS = 1.2168

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = 0.0000

ETA	F	DF	S
0.	0.	0.1428E-07	0.1000E 01
0.5000E 00	0.5264E-01	0.2342E 00	0.1000E 01
[illegible]	0.2330E 00	0.4606E 00	0.1000E 01
[illegible]	0.5150E 00	0.6615E 00	0.1000E 01
[illegible]	0.8868E 00	0.8167E 00	0.1000E 01
[illegible]	0.1322E 01	0.9168E 00	0.1000E 01
[illegible]	0.1796E 01	0.9691E 00	0.1000E 01
[illegible]	0.2286E 01	0.9907E 00	0.1000E 01
[illegible]	0.2784E 01	0.9978E 00	0.1000E 01
[illegible]	0.3283E 01	0.9996E 00	0.1000E 01
[illegible]	0.3783E 01	0.9999E 00	0.1000E 01
[illegible]	0.4283E 01	0.1000E 01	0.1000E 01
[illegible]	0.4783E 01	0.1000E 01	0.1000E 01
[illegible]	0.5283E 01	0.1000E 01	0.1000E 01
[illegible]	0.5783E 01	0.1000E 01	0.1000E 01
[illegible]	0.6283E 01	0.1000E 01	0.1000E 01

BETA = 1.0000 PRANDTLE' NO. = 5.0000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 1.4176

TEMPERATURE GRADIENT AT WALL = -0.5373

DISPLACEMENT THICKNESS = 0.8681

MOMENTUM THICKNESS = 0.2784

THERMAL THICKNESS = 0.2691

REYNOLD ANALOGY PARAMETER = 2.6387

ETA	F	DF	S
0.	0.	-0.3181E-06	0.1500E 01
0.5000E 00	0.1478E 00	0.5367E 00	0.1240E 01
0.1000E 01	0.4925E 00	0.8081E 00	0.1063E 01
0.1500E 01	0.9311E 00	0.9293E 00	0.1007E 01
0.2000E 01	0.1410E 01	0.9773E 00	0.1000E 01
0.2500E 01	0.1903E 01	0.9936E 00	0.1000E 01
0.3000E 01	0.2402E 01	0.9984E 00	0.1000E 01
0.3500E 01	0.2901E 01	0.9996E 00	0.1000E 01
0.4000E 01	0.3401E 01	0.9999E 00	0.1000E 01
0.4500E 01	0.3901E 01	0.1000E 01	0.1000E 01
0.5000E 01	0.4401E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.4901E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5401E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5901E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6401E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6901E 01	0.1000E 01	0.1000E 01

TABLE 21

BETA = 2.0000 PRANDTLE NO. = 5.0000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 1.9959

TEMPERATURE GRADIENT AT WALL = -0.5848

DISPLACEMENT THICKNESS = 0.6884

MOMENTUM THICKNESS = 0.2093

THERMAL THICKNESS = 0.2488

REYNOLD ANALOGY PARAMETER = 3.4131

ETA	F	DF	S
0.	0.	-0.3643E-05	0.1500E 01
[illegible]	0.1923E 00	0.6680E 00	0.1220E 01
0.1000E 01	0.5964E 00	0.9040E 00	0.1048E 01
0.1500E 01	0.1070E 01	0.9743E 00	0.1004E 01
0.2000E 01	0.1563E 01	0.9936E 00	0.1000E 01
0.2500E 01	0.2061E 01	0.9984E 00	0.1000E 01
0.3000E 01	0.2561E 01	0.9996E 00	0.1000E 01
0.3500E 01	0.3060E 01	0.9999E 00	0.1000E 01
0.4000E 01	0.3560E 01	0.1000E 01	0.1000E 01
0.4500E 01	0.4060E 01	0.1000E 01	0.1000E 01
0.5000E 01	0.4560E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.5060E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5560E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.6060E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6560E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.7060E 01	0.1000E 01	0.1000E 01

TABLE 22

BETA = 0.0000 PRANDTLE NO. = 9.0000 SW = 1.0000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = 0.0000

DISPLACEMENT THICKNESS = 1.2168

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = -0.0000

ETA	F	DF	S
[illegible]	0.	0.1428E-07	0.1000E 01
[illegible]	0.5864E-01	0.2342E 00	0.1000E 01
[illegible]	0.2330E 00	0.4606E 00	0.1000E 01
[illegible]	0.5150E 00	0.6615E 00	0.1000E 01
[illegible]	0.8868E 00	0.8167E 00	0.1000E 01
[illegible]	0.1322E 01	0.9168E 00	0.1000E 01
[illegible]	0.1796E 01	0.9691E 00	0.1000E 01
[illegible]	0.2286E 01	0.9907E 00	0.1000E 01
[illegible]	0.2784E 01	0.9978E 00	0.1000E 01
[illegible]	0.3283E 01	0.9996E 00	0.1000E 01
[illegible]	0.3783E 01	0.9999E 00	0.1000E 01
[illegible]	0.4283E 01	0.1000E 01	0.1000E 01
[illegible]	0.4783E 01	0.1000E 01	0.1000E 01
[illegible]	0.5283E 01	0.1000E 01	0.1000E 01
[illegible]	0.5783E 01	0.1000E 01	0.1000E 01
[illegible]	0.6283E 01	0.1000E 01	0.1000E 01

TABLE 23

BETA = 0.0000 PRANDTLE NO. = 9.0000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 0.4696

TEMPERATURE GRADIENT AT WALL = -0.4965

DISPLACEMENT THICKNESS = 1.5020

MOMENTUM THICKNESS = 0.4696

THERMAL THICKNESS = 0.2852

REYNOLD ANALOGY PARAMETER = 0.9458

ETA	F	DF	S
0.	0.	0.1428E-07	0.1500E 01
[illegible]	0.5864E-01	0.2342E 00	0.1257E 01
[illegible]	0.2330E 00	0.4606E 00	0.1075E 01
[illegible]	0.5150E 00	0.6615E 00	0.1008E 01
[illegible]	0.8868E 00	0.8167E 00	0.1000E 01
[illegible]	0.1322E 01	0.9168E 00	0.1000E 01
[illegible]	0.1796E 01	0.9691E 00	0.1000E 01
[illegible]	0.2286E 01	0.9907E 00	0.1000E 01
[illegible]	0.2784E 01	0.9978E 00	0.1000E 01
[illegible]	0.3283E 01	0.9996E 00	0.1000E 01
[illegible]	0.3783E 01	0.9999E 00	0.1000E 01
[illegible]	0.4283E 01	0.1000E 01	0.1000E 01
[illegible]	0.4783E 01	0.1000E 01	0.1000E 01
[illegible]	0.5283E 01	0.1000E 01	0.1000E 01
[illegible]	0.5783E 01	0.1000E 01	0.1000E 01
[illegible]	0.6283E 01	0.1000E 01	0.1000E 01

TABLE 24

BETA = 1.0000 PRANDTLE NO. = 9.0000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 1.3931

TEMPERATURE GRADIENT AT WALL = -0.6611

DISPLACEMENT THICKNESS = 0.8309

MOMENTUM THICKNESS = 0.2845

THERMAL THICKNESS = 0.2177

REYNOLD ANALOGY PARAMETER = 2.1072

ETA	F	DF	S
.	0.	-0.3723E-06	0.1500E 01
[illegible]	0.1450E 00	0.5268E 00	0.1187E 01
[illegible]	0.4843E 00	0.7980E 00	0.1025E 01
[illegible]	0.9191E 00	0.9242E 00	0.1001E 01
[illegible]	0.1396E 01	0.9755E 00	0.1000E 01
[illegible]	0.1889E 01	0.9932E 00	0.1000E 01
[illegible]	0.2387E 01	0.9983E 00	0.1000E 01
[illegible]	0.2887E 01	0.9996E 00	0.1000E 01
[illegible]	0.3387E 01	0.9999E 00	0.1000E 01
0.4500E 01	0.3887E 01	0.1000E 01	0.1000E 01
0.5000E 01	0.4387E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.4887E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5387E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5887E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6387E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6887E 01	0.1000E 01	0.1000E 01

TABLE 25

BETA = 2.0000 PRANDTLE NO. = 9.0000 SW = 1.5000

VELOCITY GRADIENT AT WALL = 1.9586

TEMPERATURE GRADIENT AT WALL = -0.7229

DISPLACEMENT THICKNESS = 0.6563

MOMENTUM THICKNESS = 0.2182

THERMAL THICKNESS = 0.2002

REYNOLD ANALOGY PARAMETER = 2.7093

ETA	F	DF	S
0.	0.	-0.3692E-05	0.1500E 01
0.5000E 00	0.1882E 00	0.6540E 00	0.1164E 01
0.1000E 01	0.5854E 00	0.8924E 00	0.1016E 01
0.1500E 01	0.1055E 01	0.9700E 00	0.1000E 01
0.2000E 01	0.1547E 01	0.9925E 00	0.1000E 01
0.2500E 01	0.2045E 01	0.9982E 00	0.1000E 01
0.3000E 01	0.2544E 01	0.9996E 00	0.1000E 01
0.3500E 01	0.3044E 01	0.9999E 00	0.1000E 01
0.4000E 01	0.3544E 01	0.1000E 01	0.1000E 01
0.4500E 01	0.4044E 01	0.1000E 01	0.1000E 01
0.5000E 01	0.4544E 01	0.1000E 01	0.1000E 01
0.5500E 01	0.5044E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.5544E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.6044E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.6544E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.7044E 01	0.1000E 01	0.1000E 01

TABLE 24

BETA = -0.0625 PRANDTLE NO. = 1.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.3952

TEMPERATURE GRADIENT AT WALL = 0.0908

DISPLACEMENT THICKNESS = 1.0718

MOMENTUM THICKNESS = 0.4929

THERMAL THICKNESS = -0.2507

REYNOLD ANALOGY PARAMETER = 1.7413

ETA	F	DF	S
0.	-0.	0.6726E-09	0.8000E 00
0.5000E 00	0.5041E-01	0.2035E 00	0.8453E 00
0.1000E 01	0.2046E 00	0.4132E 00	0.8893E 00
0.1500E 01	0.4619E 00	0.6123E 00	0.9289E 00
0.2000E 01	0.8112E 00	0.7776E 00	0.9603E 00
0.2500E 01	0.1231E 01	0.8926E 00	0.9813E 00
0.3000E 01	0.1695E 01	0.9574E 00	0.9927E 00
0.3500E 01	0.2182E 01	0.9863E 00	0.9977E 00
0.4000E 01	0.2679E 01	0.9965E 00	0.9994E 00
0.4500E 01	0.3178E 01	0.9993E 00	0.9999E 00
0.5000E 01	0.3678E 01	0.9999E 00	0.1000E 01
0.5500E 01	0.4177E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4677E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5177E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5677E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6177E 01	0.1000E 01	0.1000E 01

TABLE 27

BETA = -0.1250 PRANDTLE NO. = 1.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.3049

TEMPERATURE GRADIENT AT WALL = 0.0865

DISPLACEMENT THICKNESS = 1.2141

MOMENTUM THICKNESS = 0.5217

THERMAL THICKNESS = -0.2617

REYNOLD ANALOGY PARAMETER = 1.4102

[illegible]	F	DF	S
[illegible]	-0.	-0.1019E-07	0.8000E 00
[illegible]	0.4019E-01	0.1648E 00	0.8432E 00
[illegible]	0.1684E 00	0.3506E 00	0.8853E 00
[illegible]	0.3920E 00	0.5431E 00	0.9240E 00
[illegible]	0.7086E 00	0.7183E 00	0.9557E 00
[illegible]	0.1103E 01	0.8526E 00	0.9780E 00
[illegible]	0.1553E 01	0.9363E 00	0.9909E 00
[illegible]	0.2033E 01	0.9777E 00	0.9970E 00
0.4000E 01	0.2526E 01	0.9938E 00	0.9992E 00
0.4[illegible] 01	0.3025E 01	0.9986E 00	0.9998E 00
0.5000E 01	0.3524E 01	0.9998E 00	0.1000E 01
0.5500E 01	0.4024E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4524E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5024E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5524E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6024E 01	0.1000E 01	0.1000E 01

TABLE 28

BETA = -0.1875 PRANDTLE NO. = 1.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.1794

TEMPERATURE GRADIENT AT WALL = 0.0792

DISPLACEMENT THICKNESS = 1.4717

MOMENTUM THICKNESS = 0.5599

THERMAL THICKNESS = -0.2526

REYNOLD ANALOGY PARAMETER = 0.9055

ETA	F	DF	S
[illegible]	-0.	-0.1273E-07	0.8000E 00
[illegible]	0.2557E-01	0.1086E 00	0.8396E 00
[illegible]	0.1147E 00	0.2537E 00	0.8786E 00
[illegible]	0.2840E 00	0.4267E 00	0.9153E 00
[illegible]	0.5427E 00	0.6075E 00	0.9470E 00
[illegible]	0.8381E 00	0.7681E 00	0.9712E 00
[illegible]	0.1304E 01	0.8854E 00	0.9868E 00
[illegible]	0.1765E 01	0.9538E 00	0.9950E 00
[illegible]	0.2251E 01	0.9850E 00	0.9985E 00
0.4500E 01	0.2747E 01	0.9961E 00	0.9996E 00
0.5000E 01	0.3246E 01	0.9992E 00	0.9999E 00
0.5500E 01	0.3746E 01	0.9999E 00	0.1000E 01
0.6000E 01	0.4246E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.4746E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5246E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.5746E 01	0.1000E 01	0.1000E 01

TABLE 29

BETA = -0.0500 PRANDTLE NO. = 0.3000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.4149

TEMPERATURE GRADIENT AT WALL = 0.0597

DISPLACEMENT THICKNESS = 0.9015

MOMENTUM THICKNESS = 0.4840

THERMAL THICKNESS = -0.3866

REYNOLD ANALOGY PARAMETER = 2.7803

ETA	F	DF	S
0.	-0.	0.3104E-08	0.8000E 00
0.5000E 00	0.5266E-01	0.2120E 00	0.8298E 00
0.1000E 01	0.2126E 00	0.4271E 00	0.8594E 00
0.1500E 01	0.4774E 00	0.6276E 00	0.8880E 00
0.2000E 01	0.8339E 00	0.7908E 00	0.9146E 00
0.2500E 01	0.1259E 01	0.9015E 00	0.9380E 00
0.3000E 01	0.1727E 01	0.9622E 00	0.9575E 00
0.3500E 01	0.2216E 01	0.9885E 00	0.9725E 00
0.4000E 01	0.2713E 01	0.9973E 00	0.9833E 00
0.4500E 01	0.3212E 01	0.9996E 00	0.9905E 00
0.5000E 01	0.3712E 01	0.1000E 01	0.9949E 00
0.5500E 01	0.4212E 01	0.1000E 01	0.9975E 00
0.6000E 01	0.4712E 01	0.1000E 01	0.9989E 00
0.6500E 01	0.5212E 01	0.1000E 01	0.9996E 00
0.7000E 01	0.5712E 01	0.1000E 01	0.9999E 00
0.7500E 01	0.6212E 01	0.1000E 01	0.1000E 01

TABLE 3D

BETA = -0.0625 PRANDTLE NO. = 1.0000 SW = 1.0000

VELOCITY GRADIENT AT WALL = 0.3814

TEMPERATURE GRADIENT AT WALL = -0.0000

DISPLACEMENT THICKNESS = 1.3407

MOMENTUM THICKNESS = 0.4962

THERMAL THICKNESS = -0.0000

ETA	F	DF	S
0.	-0.	-0.1138E-08	0.1000E 01
0.5000E 00	0.4894E-01	0.1981E 00	0.1000E 01
0.1000E 01	0.1996E 00	0.4051E 00	0.1000E 01
0.1500E 01	0.4527E 00	0.6040E 00	0.1000E 01
0.2000E 01	0.7983E 00	0.7709E 00	0.1000E 01
0.2500E 01	0.1215E 01	0.8884E 00	0.1000E 01
0.3000E 01	0.1678E 01	0.9553E 00	0.1000E 01
0.3500E 01	0.2164E 01	0.9855E 00	0.1000E 01
0.4000E 01	0.2660E 01	0.9963E 00	0.1000E 01
0.4500E 01	0.3159E 01	0.9992E 00	0.1000E 01
0.5000E 01	0.3659E 01	0.9999E 00	0.1000E 01
0.5500E 01	0.4159E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4659E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5159E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5659E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6159E 01	0.1000E 01	0.1000E 01

TABLE 31

BETA = -0.0625 PRANDTLE NO. = 9.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.3888

TEMPERATURE GRADIENT AT WALL = 0.1889

DISPLACEMENT THICKNESS = 1.2151

MOMENTUM THICKNESS = 0.4956

THERMAL THICKNESS = -0.1196

REYNOLD ANALOGY PARAMETER = 0.8232

ETA	F	DF	S
0.	-0.	-0.1294E-08	0.8000E 00
0.5000E 00	0.4962E-01	0.2004E 00	0.8928E 00
0.1000E 01	0.2017E 00	0.4080E 00	0.9651E 00
0.1500E 01	0.4562E 00	0.6065E 00	0.9952E 00
0.2000E 01	0.8028E 00	0.7728E 00	0.9998E 00
0.2500E 01	0.1221E 01	0.8895E 00	0.1000E 01
0.3000E 01	0.1684E 01	0.9559E 00	0.1000E 01
0.3500E 01	0.2170E 01	0.9857E 00	0.1000E 01
0.4000E 01	0.2666E 01	0.9963E 00	0.1000E 01
0.4500E 01	0.3165E 01	0.9992E 00	0.1000E 01
0.5000E 01	0.3665E 01	0.9999E 00	0.1000E 01
0.5500E 01	0.4165E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4665E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.5165E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5665E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.6165E 01	0.1000E 01	0.1000E 01

TABLE 32

BETA = -0.1250 PRANDTLE NO. = 9.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.2892

TEMPERATURE GRADIENT AT WALL = 0.1752

DISPLACEMENT THICKNESS = 1.3827

MOMENTUM THICKNESS = 0.5279

THERMAL THICKNESS = -0.1285

REYNOLD ANALOGY PARAMETER = 0.6603

ETA	F	DF	S
0.	-0.	-0.1097E-07	0.8000E 00
0.5000E 00	0.3826E-01	0.1572E 00	0.8864E 00
0.1000E 01	0.1611E 00	0.3372E 00	0.9575E 00
0.1500E 01	0.3772E 00	0.5273E 00	0.9923E 00
0.2000E 01	0.6862E 00	0.7041E 00	0.9995E 00
0.2500E 01	0.1075E 01	0.8426E 00	0.1000E 01
0.3000E 01	0.1520E 01	0.9308E 00	0.1000E 01
0.3500E 01	0.1998E 01	0.9753E 00	0.1000E 01
0.4000E 01	0.2491E 01	0.9930E 00	0.1000E 01
0.4500E 01	0.2989E 01	0.9984E 00	0.1000E 01
0.5000E 01	0.3489E 01	0.9997E 00	0.1000E 01
0.5500E 01	0.3989E 01	0.1000E 01	0.1000E 01
0.6000E 01	0.4489E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.4989E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5489E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.5989E 01	0.1000E 01	0.1000E 01

TABLE 33

BETA = -0.1875 PRANDTLE NO. = 9.0000 SW = 0.8000

VELOCITY GRADIENT AT WALL = 0.1420

TEMPERATURE GRADIENT AT WALL = 0.1494

DISPLACEMENT THICKNESS = 1.7197

MOMENTUM THICKNESS = 0.5709

THERMAL THICKNESS = -0.1489

REYNOLD ANALOGY PARAMETER = 0.3802

ETA	F	DF	S
0.	-0.	-0.1074E-07	0.8000E 00
0.5000E 00	0.2094E-01	0.9025E-01	0.8742E 00
0.1000E 01	0.9677E-01	0.2194E 00	0.9406E 00
0.1500E 01	0.2460E 00	0.3822E 00	0.9832E 00
0.2000E 01	0.4818E 00	0.5617E 00	0.9980E 00
0.2500E 01	0.8058E 00	0.7300E 00	0.9999E 00
0.3000E 01	0.1205E 01	0.8601E 00	0.1000E 01
0.3500E 01	0.1657E 01	0.9406E 00	0.1000E 01
0.4000E 01	0.2139E 01	0.9797E 00	0.1000E 01
0.4500E 01	0.2633E 01	0.9945E 00	0.1000E 01
0.5000E 01	0.3132E 01	0.9988E 00	0.1000E 01
0.5500E 01	0.3631E 01	0.9998E 00	0.1000E 01
0.6000E 01	0.4131E 01	0.1000E 01	0.1000E 01
0.6500E 01	0.4631E 01	0.1000E 01	0.1000E 01
0.7000E 01	0.5131E 01	0.1000E 01	0.1000E 01
0.7500E 01	0.5631E 01	0.1000E 01	0.1000E 01

APPENDIX 2.

```
 ISN        SOURCE STATEMENT

   0 $IBFTC
   1          DIMENSION F(210),F1(210),F2(210),F3(210),S(210),S1(210),S2(21
            1  S3(210),G1(210),G2(210),G3(210),T1(210),T2(210),T3(210),
            2  AETA(210),DF(210),BS(210)
            3 ,DM(6),DN(6) ,AA(5),EIF(210),AFP(210) ,SWW(6)
   2          LOGICAL LAG
   3     4    FORMAT(/20X,27HVELOCITY GRADIENT AT WALL =,F9.4)
   4     5     FORMAT(/20X, 30HTEMPERATURE GRADIENT AT WALL =,F9.4)
   5    10      FORMAT(/25X,3HETA,12X,1HF,14X,2HDF,13X,1HS/2X)
   6     6    FORMAT(/20X,24HDISPLACEMENT THICKNESS =,F9.4)
   7     9    FORMAT(1H1,20X,6HBETA =,F9.4 ,3X, 14HPRANDTLE NO. =,F9.4 ,3X,
            1=,F9.4)
  10     7    FORMAT(/20X,20HMOMENTUM THICKNESS =,F9.4)
  11     8    FORMAT(/20X,19HTHERMAL THICKNESS =,F9.4)
  12    99    FORMAT(/20X,27HREYNOLD ANALOGY PARAMETER =,F9.4)
  13    11    FORMAT(/20X,F11.4,4X,E11.4,4X,F11.4,4X,E11.4)
  14          READ0999 ,H,DB
  15   414    FORMAT(4E15.5)
  16   999    FORMAT(F4.2,F4.2)
  17   777     FORMAT(9E12.4)
  20   555     FORMAT(F5.2,10X,     E15.5,10X,E15.5,10X,I5)
  21   419     FORMAT(3E15.5)
  22          CALL FLUN(2500)
  23          KST=1
  24          SWW(1)=0.2
  25          SWW(2)=0.6
  26          SWW(3)=1.5
  27          SWW(4)=1.
  30          SWW(5)=2.
  31          IKK=4
  32          PR=5.
  33   147    DO331MST=3,IKK
  34          JA=21
  35          NA=151
  36          A=1.
  37          AETA(1)=0.
  40          DO115I=1,2
  41          J=1
  42          B=0.
  43          Z=0.
  44          W=A
  45          ETA=0.
  46          N=1
  47          F(N)=0.
  50   222    DK0=H*Z
  51          DL0=H*W
  52          DM0=-H*(F(N)*W)
  53          DK1=H*(Z+DL0/2.)
  54          DL1=H*(W+DM0/2.)
  55          DM1=-H*((F(N)+DK0/2.)*(W+DM0/2.))
  56          DK2=H*(Z+DL1/2.)
  57          DL2=H*(W+DM1/2.)
  60          DM2=-H*((F(N)+DK1/2.)*(W+DM1/2.))
  61          DK3=H*(Z+DL2)
  62          DL3=H*(W+DM2)
```

MEG004 FORTRAN SOURCE LIST

ISN SOURCE STATEMENT

```
      DM3=-H*((F(N)+DK2)*(W+DM2))
      F(N+1)=F(N)+1./6.*(DK0+2.*DK1+2.*DK2+DK3)
      Z=Z+1./6.*(DL0+2.*DL1+2.*DL2+DL3)
      W=W+1./6.*(DM0+2.*DM1+2.*DM2+DM3)
      IF(N-NA)113,114,114
  113 ETA=ETA+H
      N=N+1
      GO TO 222
  114 A=(1./Z)**1.5
  115 CONTINUE
      CALLTICK(H,NA,F,DIF)
      II=1
      KK=1
      NK=1
      SW=SWW(MST)
      W=1.
      S(1)=SW
   56 P=W
      L=1
      DN(1)=0.
      DM(1)=0.
   55 A=0
      DO211 I=2,5
      CALL FLUN(100)
      IF(I.GT.2)A=1./2.
      IF(I.GT.4)A=1.
      FLLL=F(L)+DIF(L)*A*H
      DM(I)=(P+DN(I-1)*A)*H
      DN(I)=-H*PR*FLLL*(P+DN(I-1)*A)
  211 CONTINUE
      DDM=(1./6.)*(DM(2)+2.*DM(3)+2.*DM(4)+DM(5))
      DDN=(1./6.)*(DN(2)+2.*DN(3)+2.*DN(4)+DN(5))
      S(L+1)=S(L)+DDM
      P=P+DDN
      SS=S(L+1)
      IF(L-NA)66,77,77
   66 L=L+1
      GOTO55
   77 IF(ABS(S(NA)-1.)-.00001)92,92,88
   88 AA(II)=S(NA)
      IF(II-2)89,90,90
   89 II=2
      W=W+0.5000
      NK=NK+1
      GOTO56
   90 IF(KK.EQ.2)GOTO92
   91 IF(ABS(AA(2)-AA(1)).LT..01)GOTO89
      W=(AA(2)-W*AA(1)+W-1.)/(AA(2)-AA(1)    )
      KK=KK+1
      GOTO56
   92 DO444K=1,NA
  444 AETA(K+1)=AETA(K)+H
      R=0.
      J=1
       KKN=1
```

MEG004 FORTRAN SOURCE LIST

ISN SOURCE STATEMENT

```
      IF(KST.LT.3)GOTO2
  107 DO100M=1,NA
      F1(M)=F(M)
      S1(M)=S(M)
  100  CONTINUE
      CALLKUTTA(NA,PR,F1,S1,B,H,G1,T1,AZ,BZ,LM)
      DO101 M=1,NA
      F2(M)=F1(M)+G1(M)*DB/2.
  101 S2(M)=S1(M)+T1(M)*DB/2.
      C=B+DB/2.
      CALLKUTTA(NA,PR,F2,S2,C,H,G2,T2,AZ,BZ,LM)
      DO102M=1,NA
      F3(M)=F2(M)+G2(M)*DB/2.
  102  S3(M)=S2(M)+T2(M)*DB/2.
      D=B+DB
      CALL KUTTA(NA,PR,F3,S3,D,H,G3,T3,AZ,BZ,LM)
      DO103M=1,NA
      F(M)=DB/6.*(G1(M)+4.*G2(M)+G3(M))+F(M)
  103 S(M)=DB/6.*(T1(M)+4.*T2(M)+T3(M))+S(M)
      IF(J-JA)104,105,105
  104 B=B+DB
      J=J+1
      IF((KST.EQ.1).AND.(J.EQ.11))GOTO2
      IF((KST.EQ.1).AND.(J.EQ.21))GOTO2
       LAG=(J.EQ.2).OR.(J.EQ.6).OR.(J.EQ.8)
      IF((KST.EQ.2).AND.LAG)GOTO2
      IF((KST.EQ.2).AND.(J.GT.8))GOTO342
      IF((KST.EQ.3).AND.(J.EQ.11))GOTO2
      IF((KST.EQ.3).AND.(J.EQ.21))GOTO2
      IF((KST.EQ.4).AND.(J.EQ.11))GOTO2
      IF((KST.EQ.4).AND.(J.GT.11))GOTO342
      IF((KST.EQ.5).AND.(J.EQ.11))GOTO2
      IF((KST.EQ.5).AND.(J.GT.11))GOTO342
      GOTO107
    2 CALLTICK(H,NA,F,F1)
      DO300NT=1,NA
  300  AFP(NT)=  -F1(NT)+S(NT)
      CALL SUB(AFP,NA,H,ZA)
      DO301NT=1,NA
  301  AFP(NT)=F1(NT)*(1.-F1(NT))
      CALLSUB(AFP,NA,H,ZO)
      CALLSUB(S,NA,H,ZP)
      ZP=ZP-7.5
      HH=1./(12.*H)
      ZB=(-25.*F1(1)+48.*F1(2)-36.*F1(3)+16.*F1(4)-3.*F1(5))*HH
      ZR=(-25.*S (1)+48.*S (2)-36.*S (3)+16.*S (4)-3.*S (5))*HH
      IF(MST.NE.4)ZQ=-ZR/(SWW(MST)-1.)
      IF(MST.NE.4)ZM=2.*ZB/ZQ
      DO333  IND=1,5
      PRINT9,B,PR,SW
      PRINT4,ZB
      PRINT5,ZR
      PRINT6,ZA
      PRINT7,ZO
      PRINT8,ZP
```

MEG004 FORTRAN SOURCE LIST

```
  ISN        SOURCE STATEMENT

  314        IF(MST.NE.4)PRINT99,ZM
  317        PRINT10
  320        ETA=0.
  321        DO304N=1,NA,10
  322        PRINT11,ETA,F(N),F1(N),S(N)
  323   304   ETA=ETA+.500000
  325   333  CONTINUE
  327        GOTO107
  330   105  GOTO331
  331   331  CONTINUE
  333   342  IF(KST.EQ.2)GOTO321
  336        IF(KST.EQ.3)GOTO341
  341        IF(KST.EQ.4)GOTO351
  344        IF(KST.EQ.5)GOTO361
  347        DB=-.05
  350        KST=2
  351        IKK=3
  352        GOTO147
  353   321   DB=0.1
  354        KST=3
  355        PR=9.
  356        IKK=3
  357        GOTO147
  360   341  KST=4
  361        DB=.1
  362        PR=.3
  363        IKK=3
  364        GOTO147
  365   351  KST=4
  366        DB=.1
  367         PR=.03
  370        IKK=3
  371        GOTO147
  372   361  STOP
  373        END
```

MEG004 IBMAP ASSEMBLY

NO MESSAGES FOR ABOVE ASSEMBLY

FORTRAN SOURCE LIST

```
EG004
 IS        SOURCE STATEMENT

  $IBFTC FLICK
           SUBROUTINE FLICK(H,NA,AF,D1F,D2F)
           DIMENSION AF(210),D1F(210),D2F(210)
           CALLFLUN(800)
           NNA=NA-4
           NB=NNA+1
           HH=1./(12.*H)
           HJ=1./(12.*2.*H)
           HH1=1./(12.*(H**2))
           HJ1=1./(12.*((2.*H)**2))
           HK=(1./2.)**4
           HK1=HK-1.
           DO 46 L=1,4
           X1=(-25.*AF(L)+48.*AF(L+1)-36.*AF(L+2)+16.*AF(L+3)-3.*AF(L+4))*HH
           X2=HJ*(-25.*AF(L)+48.*AF(L+2)-36.*AF(L+4)+16.*AF(L+6)-3.*AF(L+8))
           D1F(L)=(HK*X2-X1)/HK1
           Y1=HH1*(45.*AF(L)-154.*AF(L+1)+214.*AF(L+2)-156.*AF(L+3)
          1 +61.*AF(L+4)-10.*AF(L+5))
           Y2=HJ1*(45.*AF(L)-154.*AF(L+2)+214.*AF(L+4)
          1  -156.*AF(L+6)+61.*AF(L+8)-10.*AF(L+10))
        46 D2F(L)=(HK*Y2-Y1)/HK1
           DO47L=5,NNA
           X1=HH*(AF(L-2)-8.*AF(L-1)+8.*AF(L+1)-1.*AF(L+2))
           X2=HJ*(AF(L-4)-8.*AF(L-2)+8.*AF(L+2)-1.*AF(L+4))
           D1F(L)=(HK*X2-X1)/HK1
           Y1=HH1*(-AF(L-2)+16.*AF(L-1)-30.*AF(L)+16.*AF(L+1)-AF(L+2))
           Y2=HJ1*(-AF(L-4)+16.*AF(L-2)-30.*AF(L)+16.*AF(L+2)-AF(L+4))
        47 D2F(L)=(HK*Y2-Y1)/HK1
           DO48L=NB,NA
           X1=HH*(25.*AF(L)-48.*AF(L-1)+36.*AF(L-2)-16.*AF(L-3)+3.*AF(L-4))
           X2=HJ*(25.*AF(L)-48.*AF(L-2)+36.*AF(L-4)-16.*AF(L-6)+3.*AF(L-8))
           D1F(L)=(HK*X2-X1)/HK1
           Y1=HH1*(45.*AF(L)-154.*AF(L-1)+214.*AF(L-2)-156.*AF(L-3)
          1+61.*AF(L-4)-10.*AF(L-5))
           Y2=HJ1*(45.*AF(L)-154.*AF(L-2)+214.*AF(L-4)-156.*AF(L-6)
          1  +61.*AF(L-8)-10.*AF(L-10))
        48 D2F(L)=(HK*Y2-Y1)/HK1
           RETURN
           END                                   IBMAP ASSEMBLY FLICK
MEG004

  NO MESSAGES FOR ABOVE ASSEMBLY
```

MEG004

FORTRAN SOURCE LIST

```
 ISN        SOURCE STATEMENT

   0 $IBFTC TICK
   1        SUBROUTINE TICK(H,NA,AF,D1F)
   2        DIMENSION AF(210),D1F(210)
   3        CALLFLUM(800)
   4        NNA=NA-4
   5        NB=NNA+1
   6        HH=1./(12.*H)
   7        HJ=1./(12.*2.*H)
  10        HK=(1./2.)**4
  11        HK1=HK-1.
  12        DO46L=1,4
  13        X1=(-25.*AF(L)+48.*AF(L+1)-36.*AF(L+2)+16.*AF(L+3)-3.*AF(L+4))*H
  14        X2=HJ*(-25.*AF(L)+48.*AF(L+2)-36.*AF(L+4)+16.*AF(L+6)-3.*AF(L+8)
  15    46  D1F(L)=(HK*X2-X1)/HK1
  17        DO47L=5,NNA
  20        X1=HH*(AF(L-2)-8.*AF(L-1)+8.*AF(L+1)-1.*AF(L+2))
  21        X2=HJ*(AF(L-4)-8.*AF(L-2)+8.*AF(L+2)-1.*AF(L+4))
  22    47  D1F(L)=(HK*X2-X1)/HK1
  24        DO48L=NB,NA
  25        X1=HH*(25.*AF(L)-48.*AF(L-1)+36.*AF(L-2)-16.*AF(L-3)+3.*AF(L-4))
  26        X2=HJ*(25.*AF(L)-48.*AF(L-2)+36.*AF(L-4)-16.*AF(L-6)+3.*AF(L-8))
  27    48  D1F(L)=(HK*X2-X1)/HK1
  31        RETURN
  32        END
```

MEG004

IBMAP ASSEMBLY TICK

NO MESSAGES FOR ABOVE ASSEMBLY

ISN SOURCE STATEMENT

```
$IBFTC KUTTA
      SUBROUTINE KUTTA(NA,PR,AF,S,B,H,G,T,P,TT,KK)
      LOGICAL LAG
      DIMENSIONAF(210),S(210),D1F(210),D2F(210),D3F(210),G(210),T(210)
     1D1S(210),D2S(210),AV(9),AVV(9)
     2 ,                DK(6),DL(6),DM(6),DN(6),DO(6),AT(20),AP(20)
  420 FORMAT(4E15.5)
      CALL FLUM(3000)
      CALLFLICK(H,NA,AF,D1F,D2F)
      CALLTICK(H,NA,S,D1S)
      DO11 L=1,NA
      D3F(L)=-(AF(L)*D2F(L)+B*(S(L)-D1F(L)**2))
   11 D2S(L)=-PR*(AF(L)*D1S(L))
      DK(1)=0.
      AV(1)=1.
      AVV(1)=1.
      AV(2)=1.2
      AVV(2)=1.
      AV(3)=1.
      AVV(3)=1.3
       AV(4)=1.2
      AVV(4)=1.3
      AV(5)=1.6
      AVV(5)=1.2
      AV(6)=1.8
      AVV(6)=1.2
      AV(7)=1.6
      AVV(7)=1.5
      AV(8)=2.
      AVV(8)=2.1
      NS=3
      DL(1)=0.
      DM(1)=0.
      DN(1)=0.
      DO(1)=0.
      II=1
      NK=1
      KK=1
      W=1.
      WW=1.
   38 L=1
      G(1)=0
      T(1)=0.
      R=WW
      P=0.
      Q=W
   33 DO444I=2,5
      CALL FLUM(100)
      A=0.
      IF(I.GT.2)A=1./2.
      IF(I.GT.4)A=1.
      A1=D1F(L)+A*D2F(L)*H
      A2=D2F (L)+A*D3F(L)*H
      A3=D1S(L)+A*D2S(L)*H
      A4=AF(L)+D1F(L)*H*A
```

```
ISN         SOURCE STATEMENT

            A5=S(L)+DTS(L)*H*A
            X=G(L)+DK(I-1)*A
            Y=P+DL(I-1)*A
            Z=Q+DM(I-1)*A
            U=T(L)+DN(I-1)*A
            V=R+DO(I-1)*A
            DK(I)=H*Y
             DL(I)=H*Z
            DM(I)=-H*(X*A2+A4*Z+B*(U-2.*A1*Y)+(A5-A1**2))
            DN(I)=H*V
            DO(I)=-H*PR*(A4*V+A3*X)
     444    CONTINUE
            G(L+1)=G(L)+1./6.*(DK(2)+2.*DK(3)+2.*DK(4)+DK(5))
                     P=P+1./6.*(DL(2)+2.*DL(3)+2.*DL(4)+DL(5))
            Q=Q+1./6.*(DM(2)+2.*DM(3)+2.*DM(4)+DM(5))
            T(L+1)=T(L)+1./6.*(DN(2)+2.*DN(3)+2.*DN(4)+DN(5))
            R=R+1./6.*(DO(2)+2.*DO(3)+2.*DO(4)+DO(5))
            TL=T(L+1)
            GG=G(L+1)
            IF(L-NA)31,32,32
      31    L=L+1
            GOTO33
      32     TT=T(NA)
            LAG=ABS(P).LT..00001.AND.TT.LT..00001
            IF(LAG)GOTO35
            AT(II)=TT
            AP(II)=P
            IF(II-NS)36,37,37
      36     II=1+II
            W=AV(II)
            WW=AVV(II)
            GOTO38
      37    IF(KK.EQ.2)GOTO35
            C1=(AT(2)-AT(1))/( AV(2)-AV(1))
            C2=(AP(2)-AP(1))/ (AV(2)-AV(1))
            C3=(AT(3)-AT(1))/(AVV(3)-AVV(1))
            C4=(AP(3)-AP(1))/(AVV(3)-AVV(1))
            DFP=-AP(1)
            DFT=-AT(1)
            XC=C1*C4-C2*C3
            DAV=(C4*DFT-DFP*C3)/XC
            DAVV=(C1*DFP-C2*DFT)/XC
            W=AV(1)+DAV
            WW=AVV(1)+DAVV
            KK=KK+1
            GOTO38
      35     RETURN
            END
```

ME0004 IBMAP ASSEMBLY KUTTA

NO MESSAGES FOR ABOVE ASSEMBLY

```
MEG004
     ISN          SOURCE STATEMENT                              FORTRAN SOURCE LIST

        0  $IBFTCSUB
        1           SUBROUTINE SUB(Y,M,H,AREA)
        2           DIMENSION Y(210)
        3           SUMEV=0.
        4           SUMOD=0.
        5           N=M-1
        6           K=N-1
        7           DO4I=2,N,2
       10     4     SUMEV=SUMEV+Y(I)
       12           DO5I=3,K,2
       13      5     SUMOD=SUMOD+Y(I)
       15           AREA=H/3.*(Y(1)+4.*SUMEV+2.*SUMOD+Y(M))
       16           RETURN
       17           END
MEG004
                                                                IBMAP ASSEMBLY UB

   NO MESSAGES FOR ABOVE ASSEMBLY
MEG004
                                                                IBLDR -- JOB   000000

PROGRAM IS BEING ENTERED INTO STORAGE.
```

APPENDIX 3

```
 ISN       SOURCE STATEMENT

   0 $IBFTC
   1          DIMENSION ST(15),FN(10),JCI(25),JCF(25),JQ(25),C(25),
             1AF(160,25),Q(160,25),AETA(160),JEQ(25),
             2JC(6),            NNV(10),SST(15,6),FFN(10,6),JJCI(25,6),JJCF(25,6),
             3JJEQ(25,6)     ,    LCH(6),NIB(6),NFB(6)
   2          LOGICAL  LG
   3   301     FORMAT(5X,39HSOLUTION OBTAINED WITH PARA.DIFF.METHOD)
   4   507     FORMAT(2X,6HAF(3)=,E13.4,6HAF(6)=,E14.4)
   5   302     FORMAT(5X,45HSOLUTION OBTAINED DIRECTLY FROM ORIGINAL EQN.)
   6   300     FORMAT(5X,10HPARAMETER=,F11.4)
   7   100    FORMAT(F4.2,F4.2,2I3)
  10    209   FORMAT(I1)
  11    904     FORMAT(8E11.4)
  12    23      FORMAT(4HKMP=,I3)
  13   14     FORMAT(2F5.3)
  14   15     FORMAT(7I1)
  15    16      FORMAT(3I1)
  16   13      FORMAT(3F5.3)
  17   11      FORMAT(3I2)
  20   10      FORMAT(2I3,F5.3 )
  21   117    FORMAT(F5.3,I3)
  22          READ117,DB,JA
  24          READ10,NKMP,NR,H
  27          READ11,(NNV(I),I=1,NKMP)
  34          READ11,(NIB(I),I=1,NKMP)
  41          READ11,(NFB(I),I=1,NKMP)
  46          DO12I=1,NKMP
  47          M=NIB(I)
  50          MN=NFB(I)
  51          N=NNV(I)
  52          READ13,(SST(K,I),K=1,M)
  57          READ14,(FFN(K,I),K=1,MN)
  64          READ15,(JJCI(K,I),K=1,N)
  71          READ15,(JJCF(K,I),K=1,N)
  76          READ15,(JJEQ(K,I),K=1,N)
 103    12    CONTINUE
 105          READ16,(LCH(K),K=1,NKMP)
 112          READ209,MI
 114          AETA(1)=0.
 115           NAA=NR-1
 116          DO444K=1,NAA
 117   444    AETA(K+1)=AETA(K)+H
 121          DO17KMP=1,NKMP
 122          LG=((LCH(KMP).EQ.2).OR.(LCH(KMP).EQ.3))
 123          NV=NNV(KMP)
 124          IBB=NIB(KMP)
 125          IFB=NFB(KMP)
 126          DO18J=1,NV
 127          JCI(J)=JJCI(J,KMP)
 130          JEQ(J)=JJEQ(J,KMP)
 131    18    JCF(J)=JJCF(J,KMP)
 133          DO19J=1,IBB
 134   19       ST(J)=SST(J,KMP)
 136          DO200J=1,IFB
 137   200    FN(J)=FFN(J,KMP)
```

```
G004
                                          FORTRAN SOURCE LIST
  ISN        SOURCE STATEMENT

  141           CALL ANT(NV,JEQ,JQ,IC,MI,JC,JCF)
  142           LLM=LCH(KMP)
  143           IF(LCH(KMP).EQ.0)CALLBST(AF,NR,H)
  146           IF(LCH(KMP).EQ.1)CALL MIST(JA,DB,H,B,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,
             1IC,MI,KMP,AF,Q,C,JC,LG)
  151           IF(LCH(KMP).EQ.2)CALL HIX(H,B,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,IC,MI,
             2KMP,AF,Q,C,JC,LG,LLM)
  154           IF(LCH(KMP).EQ.3)GOTO20
  157           IF(LCH(KMP).EQ.0)GOTO17
  162           IF(LCH(KMP).EQ.1)GOTO21
  165           IF(LCH(KMP).EQ.2)GOTO22
  170     22    PRINT302
  171           PRINT23,KMP
  172           GOTO304
  173     20    DO24NS=1,NR
  174           DO24NJ=1,NV
  175     24     Q(NS,NJ)=AF(NS,NJ)
  200            CALL HIX(H,B,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,IC,MI,KMP,AF ,Q,C,JC
             1,LG,LLM)
  201           GOTO304
  202     21    PRINT301
  203           PRINT300,B
  204    304    DO805 N=1,NV
  205           IF(N.EQ.1)          PRINT904,(AF(I,N)   ,AETA(I),I=1,NR)
  214           IF(JEQ(N).EQ.2) PRINT904,(AF(I,N)   ,AETA(I),I=1,NR)
  223    805    CONTINUE
  225           GOTO17
  226     17     CONTINUE
  230           STOP
  231           END
004
                                          IBMAP ASSEMBLY

NO MESSAGES FOR ABOVE ASSEMBLY
```

ISN SOURCE STATEMENT

```
  $IBFTC MIST
        SUBROUTINEMIST (JA,DB,H,B,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,IC,MI,KMP,AF
       1,Q,C,JC,LG)
        LOGICAL LG
        DIMENSION      JCF(25),JEQ(25),JCI(25),JQ(25),JC(6),AF(160,25),Q(1
       260,25),Q1(160,25),AFF(160),D1F(160),C(20) ,AF1(160,25),ST(25),FN(
       310)
        DO909I=1,IC
  909    C(I)=1.0000
        NVV=NV-1
        DO 512 I=1,NVV
        IF(JQ(I).EQ.1)GOTO512
        DO513N=1,NR
  513   AFF(N)=AF(N,I)
        CALL TICK(H,NR,AFF,D1F)
        DO514N=1,NR
  514   AF(N,I+1)=D1F(N)
  512   CONTINUE
        XYZ=DB/6.
        DBB=DB/2.
        B=0.
        JJ=1
  807   DO801I=1,NR
        DO801N=1,NV
  801   AF1(I,N)=AF(I,N)
        D=B
        AA=.1
        CALL RKM(H,D,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,IC,MI,KMP,AF1,Q,C,JC,AA
       1,LG)
        DO802N=1,NV
        DO802I=1,NR
        CALL FLUN(3000)
        AF1(I,N)=AF(I,N)+Q(I,N)*DBB
  802   CONTINUE
        D=B+DBB
        CALL RKM(H,D,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,IC,MI,KMP,AF1,Q1,C,JC,AA
       1,LG)
        DO803N=1,NV
        DO803I=1,NR
        CALL FLUN(3000)
                          AF1(I,N)=AF1(I,N)+Q1(I,N)*DBB
                           Q(I,N)=Q(I,N)+4.*Q1(I,N)
  803   CONTINUE
        D=B+DB
        CALL RKM(H,D,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,IC,MI,KMP,AF1,Q1,C,JC,AA
       1,LG)
        DO804 N=1,NV
        DO804 I=1,NR
                             Q(I,N)=Q(I,N)+Q1(I,N)
                             AF(I,N)=AF(I,N)+XYZ*Q(I,N)
   804 CONTINUE
   904 FORMAT(6E15.5)
        B=B+DB
        JJ=JJ+1
        IF(JJ.EQ.JA)GOTO808
```

80 93

```
004                                                     FORTRAN SOURCE LIST MIST
  ISN         SOURCE STATEMENT

  101         GOTO807
  102   808    RETURN
  103         END
004                                                     IBMAP ASSEMBLY MIST

NO MESSAGES FOR ABOVE ASSEMBLY.
```

```
6004                                                 FORTRAN SOURCE LIST
  ISN        SOURCE STATEMENT

    0 $IBFTC HIX
    1         SUBROUTINE HIX( H,D,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,IC, MI,KMP,AF,Q,C,J
             1C,LG,LM)
    2         LOGICAL LG
    3         DIMENSIONST(25),JCF(25),JEQ(25),JCI(25),JQ(25),JC(6),AF(160,25),
             1Q(160,25),FN(10),C(20)
    4         IF(LM.EQ.2)GOTO3
    7         GOTO4
   10     3   NN=1
   11         DO1I=1,NV
   12         IF(JCI(I).EQ.1)GOTO2
   15         GOTO1
   16     2   C(NN)=AF(1,I)
   17         NN=NN+1
   20     1   CONTINUE
   22     4   AA=.01
   23         CALL RKM( H,B,NV,NR,ST,FN,JCF,JEQ,JCI,JQ,IC,MI,KMP,Q,AF,C,JC,AA
            1,LG)
   24         RETURN
   25         END
004                                                 IBMAP ASSEMBLY HIX

NO MESSAGES FOR ABOVE ASSEMBLY
```

```
EG004                                                   FORTRAN SOURCE LIST
  ISN          SOURCE STATEMENT

    0 $IBFTC ANT
    1          SUBROUTINE  ANT(NV,JEQ,JQ,IC,MI,JC,JCF)
    2          DIMENSIONJEQ(25),JQ(25),JC(6),JCF(25)
    3          DO8I=1,NV
    4     8    JQ(I)=JEQ(I)
    6          NS=NV-1
    7          DO7 I=1,NS
   10          N=I+1
   11          IF(JEQ(I).EQ.1)JEQ(N)=2
   14     7    CONTINUE
   16          DO117N=1,5
   17   117     JC(N)=0
   21          IF(MI.EQ.0)GOTO116
   24          DO11N=1,MI
   25    11     JC(N)=1
   27    116    IA=0
   30          DO9I=1,NV
   31          IF(JCF(I).EQ.0)GOTO9
   34          IA=IA+1
   35     9    CONTINUE
   37          IC=IA
   40          RETURN
   41          END
EG004                                                   IBMAP ASSEMBLY ANT


 NO MESSAGES FOR ABOVE ASSEMBLY
```

ISN SOURCE STATEMENT

```
 $IBFTC RKM
       SUBROUTINERKM(H,B,NV,NR,S,FN,JCF,JEQ,JCI,JQ,IC,MI,KMP,Q,F,C,JC,A
      1,LG)
       LOGICAL LG
       LOGICAL LAG,LAD(6)
       DIMENSIONS(25),FN(10),JCF(25),JEQ(25),JCI(25),JQ(25),F(160,25),
      1Q(160,25),BQ(5),D(11,11),C(20),G(11),DF(5,25),FF(160),DFF(160),
      2Y(11,11),T(11),LA(11,11),CY(11,11),X(11),JC(6)    ,LSUM(11)
  200   FORMAT(E14.4)
  212  FORMAT(2X,4HG(I),E14.4)
  211   FORMAT(4I3)
  213  FORMAT(2X,I3, 7HF(I,IZ),2I3)
  214  FORMAT(2X,2HDF,E14.4,2I3)
  215  FORMAT(2X,8HY(J,NN)=,E14.4,4I3)
  216  FORMAT(2 X,9HLA(IP,IN),E14.4,2I3)
  217  FORMAT(2X,8HLSUM(IP),2I3)
  218   FORMAT(2X, 8HCY(M,LL),E14.4,2I3)
  219  FORMAT(2X,8HCY(I,JK),E14.4,2I3)
  220   FORMAT(2X,4HX(I),2E14.4)
  221  FORMAT(2X,4HC(I),2E14.4 )
  222  FORMAT(2X,2HI=,I3)
  400   FORMAT(1X,2HDF,7E14.4)
   33   FORMAT(2F5.2)
   32  FORMAT(3F5.2)
   31     FORMAT(10X,7I2)
       CALLFLUN(3000)
       PRINT31,(JEQ(I),I=1,NV)
       PRINT31,(JCF(I),I=1,NV)
       PRINT31,(JCI(I),I=1,NV)
       PRINT31,( JQ(I),I=1,NV)
       PRINT32,S(1),S(2),S(3)
       PRINT33,FN(1),FN(2)
       PRINT200,A
       PRINT211,KMP,NV,IC,MI
       BQ(1)=.1
       BQ(2)=.001
       BQ(3)=.0001
       BQ(4)=.00001
       BQ(5)=.000001
       PRINT221,C(1),C(2)
       ISUM=0
  461  Z=A*2.
       KK=1
       KKK=1
       IF(IC.EQ.0)GOTO418
       DO10 I=1,IC
       DO10 J=1,IC
       IF(I.EQ.J)D(I,J)=Z
       IF(I.NE.J)D(I,J)=0.
   10  CONTINUE
  418  JK=IC+1
       KK=1
       KKK=1
       ZU=1.00000/6.00000
  100   DO320 J=1,JK
```

ISN SOURCE STATEMENT

```
      CALLFLUN(3000)
      IF(IC.EQ.0)GOTO40
      DO11 I=1,IC
      IF(J.EQ.1)G(I)=C(I)
      IF(J.GT.1)G(I)=C(I)+D(J-1,I)
 11   CONTINUE
      PRINT221,G(1),G(2)
 40     NS=1
      NB=1
      DO12 IZ=1,NV
      IF(JQ(IZ).EQ.1)GOTO15
      IF(JCI(IZ).EQ.0)GOTO13
      IF(JCI(IZ).EQ.1)F(1,IZ)=G(NB)
      NB=NB+1
      GOTO12
 13   F(1,IZ)=S(NS)
      NS=NS+1
      GOTO12
 15   F(1,IZ)=0.
 12    CONTINUE
      K=1
      DO19 M=1,NV
 19   DF(1,M)=0
      DO547M=1,5
 547  DF(M,1)=0.
 21   DO20 M=2,5
      NO=0
      DO20 MA=2,NV
      CALLFLUN(500)
      IF(M.EQ.2)A=0.
      IF(M.EQ.3)A=0.50000
      IF(M.EQ.4)A=.50000
      IF(M.EQ.5)A=1.00000
      IF(JEQ(MA).EQ.0)DF(M,MA)= H*(DF(M-1,MA+1)*A+F(K,MA))
      IF(JEQ(MA).EQ.1)GOTO501
      IF(JEQ(MA).EQ.2) DF(M,MA)=DF(1,MA)
      GOTO20
 501   NO=NO+1
       CALL FUNSON(H,B,A,K,NV,JQ,M,MA,Q,F,DF,DFP,KMP,NO)
      DF(M,MA)=DFP
 20   CONTINUE
      DO739M=1,NV
      CALLFLUN(500)
      IF(JQ(M).EQ.0)F(K+1,M)=F(K,M)+ZU*(DF(2,M+1)+2.*DF(3,M+1)
     1  +2.*DF(4,M+1)+DF(5,M+1))
      IF(JQ(M).EQ.1)F(K+1,M)=0.
 739  CONTINUE
      K=K+1
      IF(K.EQ.NR)GOTO300
      GOTO21
 300  NN=1
      IF(IC.EQ.0)GOTO410
      DO23 M=1,NV
      IF(JCF(M).EQ.0)GOTO23
      IF(JCF(M).EQ.1)Y(J,NN)=F(NR,M)
```

```
 ISN      SOURCE STATEMENT

 257           PRINT215,Y(J,NN),J,NN,NR,M
 260           NN=NN+1
 261     23    CONTINUE
 263           DO401 IP=1,5
 264           DO401 IN=1,IC
 265           T(IN)=ABS(Y(J,IN)-FN(IN))
 266           LAG=T(IN).LT.BQ(IP)
 267           IF(LAG) LA(IP,IN)=1
 272           IF(.NOT.LAG)LA(IP,IN)=0
 275    401     CONTINUE
 300           DO402 IP=1,5
 301           ISUM=0
 302           DO402 IN=1,IC
 303           ISUM=LA(IP,IN)+ISUM
 304           LSUM(IP)=ISUM
 305    402    CONTINUE
 310           DO403 IP=1,5
 311    403    LAD(IP)=LSUM(IP).EQ.IC
 313           IF(LAD(5))GOTO410
 316    320     CONTINUE
 320           LAG=KK.EQ.2
 321           IF((.NOT.LAD(1)).AND.(LAG). AND.(JC(1).EQ.1))GOTO460
 324           IF((.NOT.LAD(2)).AND.(LAG). AND.(JC(2).EQ.1))GOTO460
 327           IF((.NOT.LAD(3)).AND.(LAG). AND.(JC(3).EQ.1))GOTO462
 332           IF((.NOT.LAD(4)).AND.(LAG).AND.(JC(4).EQ.1))GOTO462
 335           IF(LAD(5).AND.(JC(5).EQ.1))GOTO410
 340           IF((.NOT.LAD(5)).AND.(LAG).AND.(JC(5).EQ.1))GOTO463
 343           IF(LAD(4).AND.(JC(4).EQ.1))GOTO410
 346           IF(KK.EQ.2)GOTO410
 351           DO25 L=2,JK
 352           DO25M=1,IC
 353           LL=L-1
 354           CY(M,LL)=(Y(L,M)-Y(1,M))/Z
 355           PRINT218,CY(M,LL),M,LL
 356     25    CONTINUE
 361           DO26 I=1,IC
 362           CY(I,JK)=FN(I)-Y(1,I)
 363           PRINT219,CY(I,JK),I,JK
 364     26       CONTINUE
 366           CALL ALEQ(JK,CY,X)
 367           PRINT220,(X(I),I=1,IC)
 374           PRINT221,(C(I),I=1,IC)
 401           DO70I=1,IC
 402           C(I)=C(I)+X(I)
 403     70     CONTINUE
 405           PRINT221,(C(I),I=1,IC)
 412           KK=KK+1
 413           JK=1
 414           GOTO100
 415    410    DO4I=2,NV
 416           IF(JC(I).EQ.1) GOTO5
 421           GOTO4
 422      5    DO6N=1,NR
 423      6    FF(N)=F(N,I-1)
 425           CALL TICK(H,NR,FF,DFF)
```

EG004 FORTRAN SOURCE LIST RKM

```
   ISN         SOURCE STATEMENT

   426         DO7N=1,NR
   427         F(N,I)=DFF(N)
   430    7    CONTINUE
   432    4     CONTINUE
   434         GOTO479
   435   460   IF(LG)A=.005
   440         A=0.025
   441         GOTO461
   442   462   IF(LG)A=.002
   445         A=.005
   446         GOTO461
   447   463   IF(LG)A=.001
   452         A=.002
   453         GOTO461
   454   479   RETURN
   455         END
```

EG004 IBMAP ASSEMBLY RKM

NO MESSAGES FOR ABOVE ASSEMBLY

```
EG004                                            FORTRAN SOURCE LIST
  ISN       SOURCE STATEMENT

    0 $IBFTC ALEQ
    1           SUBROUTINE ALEQ(M,A,X)
    2           DIMENSIONA(11,11),B(11,11),X(11)
    3           CALLFLUN(500)
    4           N=M-1
    5   5        IF(A(1,1))11,6,11
    6    6      K=M-1
    7           DO9I=2,K
   10           IF(A(I,1))7,9,7
   11        7 DO8J=1,M
   12           TEMP=A(I,J)
   13           A(I,J)=A(1,J)
   14    8      A(1,J)=TEMP
   16           GOTO11
   17    9      CONTINUE
   21           GOTO18
   22    11     DO12J=2,M
   23           DO12I=2,N
   24    12     B(I-1,J-1)=A(I,J)-A(1,J)*A(I,1)/A(1,1)
   27           DO13J=2,M
   30    13      B(N,J-1)=A(1,J)/A(1,1)
   32           M=M-1
   33           DO14J=1,M
   34           DO14I=1,N
   35    14     A(I,J)=B(I,J)
   40           IF(M-1)5,16,5
   41    16     DO17I=1,N
   42    17       X(I)=A(I,1)
   44    18     RETURN
   45           END
MEG004                                           IBMAP ASSEMBLY ALEQ

 NO MESSAGES FOR ABOVE ASSEMBLY
```

EG004 FORTRAN SOURCE LIST

```
ISN       SOURCE STATEMENT

   $IBFTC TICK
          SUBROUTINE TICK(H,NA,AF,D1F)
          DIMENSION AF(210),D1F(210)
          CALLFLUN(3000)
          NNA=NA-4
          NB=NNA+1
          HH=1./(12.*H)
          HJ=1./(12.*2.*H)
          HK=(1./2.)**4
          HK1=HK-1.
          DO46L=1,4
          X1=(-25.*AF(L)+48.*AF(L+1)-36.*AF(L+2)+16.*AF(L+3)-3.*AF(L+4))*HH
          X2=HJ*(-25.*AF(L)+48.*AF(L+2)-36.*AF(L+4)+16.*AF(L+6)-3.*AF(L+8))
      46  D1F(L)=(HK*X2-X1)/HK1
          DO47L=5,NNA
          X1=HH*(AF(L-2)-8.*AF(L-1)+8.*AF(L+1)-1.*AF(L+2))
          X2=HJ*(AF(L-4)-8.*AF(L-2)+8.*AF(L+2)-1.*AF(L+4))
      47  D1F(L)=(HK*X2-X1)/HK1
          DO48L=NB,NA
          X1=HH*(25.*AF(L)-48.*AF(L-1)+36.*AF(L-2)-16.*AF(L-3)+3.*AF(L-4))
          X2=HJ*(25.*AF(L)-48.*AF(L-2)+36.*AF(L-4)-16.*AF(L-6)+3.*AF(L-8))
      48  D1F(L)=(HK*X2-X1)/HK1
          RETURN
          END
```

EG004 IBMAP ASSEMBLY TICK

NO MESSAGES FOR ABOVE ASSEMBLY

```
EG004                                           FORTRAN SOURCE LIST
   ISN        SOURCE STATEMENT

     0 $IBFTC FUNSON
     1          SUBROUTINE FUNSON (H,B,A,K,NV,JQ,M,MA,FF,QQ,DF,DFF,KMP,N)
     2          DIMENSIONJQ(25),F(25),Q(25),DF(5,25),QQ(160,25),FF(160,25)
     3          NS=NV-1
     4          CALLFLUN(500)
     5           DO900I=1,NS
     6          IF(JQ(I).EQ.1)GOTO900
    11          IF(JQ(I).EQ.0)Q(I)=QQ(K,I)+DF(M-1,I+1)*A
    14          F(I)= FF(K,I)+FF(K,I+1)*A *H
    15    900   CONTINUE
    17          GOTO(100,200,300),KMP
    20    200   GOTO(201,202),N
    21    300   GOTO(301,302),N
    22    201   FOMULA=-(F(1)*F(3)+F(5)-F(2)**2+B*(Q(1)*F(3)+F(1)*Q(3)+Q(5)-2.*F(
               12)*Q(2))-F(3)+Q(3)*(1.-B))
    23          GOTO11
    24    202   FOMULA=-(F(1)*F(6)+B*Q(1)*F(6)+B*F(1)*Q(6)+Q(6)-B*Q(6)-F(6))
    25          GOTO11
    26    301    FOMULA=-(Q(1)*Q(3)+Q(5)-Q(2)**2)
    27          GOTO11
    30    302    FOMULA=-Q(1)*Q(6)
    31          GOTO11
    32    100    CONTINUE
    33     11    DFF=H*FOMULA
    34          RETURN
    35          END
EG004                                           IBMAP ASSEMBLY FUNSON


 NO MESSAGES FOR ABOVE ASSEMBLY
```

MEG004 FORTRAN SOURCE LIST

```
      ISN       SOURCE STATEMENT

        0 $IBFTC BST
        1         SUBROUTINE BST(AF,NR,H)
        2         DIMENSION AF(160,25)
        3         ETA=0.
        4         DO221I=1,NR
        5         AF(I,1)=ETA+EXP(-ETA)-1.
        6         AF(I,5)=1.+(1.5-1.)*EXP(-ETA)
        7     221 ETA=ETA+H
       11         RETURN
       12         END
```

MEG004 IBMAP ASSEMBLY BST

NO MESSAGES FOR ABOVE ASSEMBLY

MEG004 IBLDR -- JOB 000006

M E M O R Y   M A P

```
 INCLUDING IOCS                                   00000 THRU  12211

OCK ORIGIN                                        12220

JMBER OF FILES -        2.

.  S.FBIN                          12220
.  S.FBOU                          12243

 PROGRAM                                          12266 THRU  71713

.  DECK '      '                   12266
.  DECK 'MIST  '                   35051
.  DECK 'HIX   '                   56135
.  DECK 'ANT   '                   56342
.  DECK 'RKM   '    *              56546
.  DECK 'ALEQ  '                   63161
.  DECK 'TICK  '    *              63704
.  DECK 'FUNSON'    *              64371
.  DECK 'BST   '    *              65025
).  SUBR 'INSYFB'                  65140
1.  SUBR 'OUSYFB'                  65177
2.  SUBR 'POSTX '                  65230
3.  SUBR 'CNSTNT'                  65542
4.  SUBR 'FPR   '                  65551
5.  SUBR 'FRD   '                  65552
6.  SUBR 'IOS   '                  65553
7.  SUBR 'RWD   '                  66032
8.  SUBR 'ECV   '                  67206
9.  SUBR 'FCV   '                  67454
0.  SUBR 'HCV   '                  67546
21. SUBR 'ICV   '                  67651
22. SUBR 'XCV   '                  67671
23. SUBR 'INTJ  '                  67707
24. SUBR 'FFC   '    *             70223
25. SUBR 'FPT   '                  70645
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